

Further Pure 1 The t- formulae

Solve $8 \cot x + 4 \operatorname{cosec} x + 1 = 0$

$$t = \tan\left(\frac{x}{2}\right) \Rightarrow \tan x = \frac{2t}{1-t^2} \text{ and } \sin x = \frac{2t}{1+t^2}$$

$$8\left(\frac{1-t^2}{2t}\right) + 4\left(\frac{1+t^2}{2t}\right) + 1 = 0 \Rightarrow$$

$$8 - 8t^2 + 4 + 4t^2 + 2t = 0 \Rightarrow$$

$$4t^2 - 2t - 12 = 0 \Rightarrow$$

$$2t^2 - t - 6 = 0 \Rightarrow$$

$$(2t + 3)(t - 2) = 0$$

$$\tan\frac{x}{2} = -\frac{3}{2} \text{ or } \tan\frac{x}{2} = 2$$

$$\frac{x}{2} = k\pi + \arctan\left(-\frac{3}{2}\right) \text{ or } \frac{x}{2} = k\pi + \arctan(2), \quad k \in \mathbb{Z}$$

$$x = 2k\pi + 2\arctan\left(-\frac{3}{2}\right) \text{ or } x = 2k\pi + 2\arctan(2)$$