Find $I = \int \sinh x \sin ax \, dx$

$$I = \cosh x \sin ax - a \int \cosh x \cos ax \, dx$$

$$I = \cosh x \sin ax - a \left(\sinh x \cos ax + a \int \sinh x \sin ax \, dx \right)$$

$$I = \cosh x \sin ax - a \sinh x \cos ax - a^2 I$$

$$I = \frac{\cosh x \sin ax - a \sinh x \cos ax}{1 + a^2} + C$$

Find $\int_{-\pi}^{\pi} \sinh x \sin nx \, dx$

$$\int_{-\pi}^{\pi} \sinh x \sin nx \, dx = \left[\frac{\cosh x \sin nx - n \sinh x \cos nx}{1 + n^2} \right]_{-\pi}^{\pi}$$

$$= \left(\frac{\cosh \pi \quad \sin n\pi - n \sinh \pi \cos a\pi}{1 + n^2} \right) - \left(\frac{-\cosh \pi \sin n\pi + n \sinh \pi \cos n\pi}{1 + n^2} \right)$$

$$= \frac{2 \cosh \pi \sin n\pi - 2n \sinh \pi \cos n\pi}{1 + n^2}$$

Find the fourier series for $f(x) = \sinh x$ defined on the interval $[-\pi, \pi]$.

$\sinh x$ is an odd function therefore the series consists only of sine terms.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

where

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \sinh x \sin nx \, dx = \frac{2 \cosh \pi \sin n\pi - 2n \sinh \pi \cos n\pi}{\pi(1 + n^2)}$$

$$f(x) = \sum_{n=1}^{\infty} \left(\left(\frac{2 \cosh \pi \sin n\pi - 2n \sinh \pi \cos n\pi}{\pi(1 + n^2)} \right) \sin nx \right)$$

$$= -\frac{2 \sinh \pi}{\pi} \sum_{n=1}^{\infty} \frac{n(-1)^n \sin nx}{1 + n^2}$$

https://www.desmos.com/calculator/e58lmhayxd