1)

$$x - 3y + 1 = 0 \Rightarrow y = \frac{x+1}{3}$$

$$3x^2 - 7x\left(\frac{x+1}{3}\right) - 5 = 0$$

$$2x^2 - 7x - 15 = 0$$

$$\sum x = \frac{7}{2}$$

D

2)

$$1 - c^2 + 3c - 3 = 0$$

$$c^2 - 3c + 2 = 0$$

$$(c-1)(c-2) = 0$$

$\cos\theta = 1 \Rightarrow \theta = 0, 2\pi, 4\pi$ 3 solutions

D

3)

$$y + 1 = \frac{5-2}{-6-4}\left(x - \frac{7}{2}\right)$$

$$y = 0 \Rightarrow x = -\frac{10}{3} + \frac{7}{2} = \frac{1}{6}$$

B

4)

$(x+1)(x-1)(x-2) > 0 \Rightarrow -1 < x < 1$ or $x > 2$

E

5)

$$y = -\log_{10}(1-x) \Rightarrow 10^y = \frac{1}{1-x} \Rightarrow 1 - x = 10^{-y} \Rightarrow x = 1 - 10^{-y}$$

D

6)

$$(-2)^3 + 16c - 2(c^2 + 2c + 1) - 6 = 0$$

$$-2c^2 + 12c - 16 = 0$$

$$c^2 - 6c + 8 = 0$$

$$\sum c = 6$$

D

7)

Probability second ball is not the same as the first $= \frac{2n}{3n-1}$

C

8)

$$x \log a + 2x \log b + 3x \log c = \log 2$$

$$x(\log ab^2c^3\ ) = \log 2$$

$$x = \frac{\log 2}{\log(ab^2c^3)}$$

F

9)

$$2\alpha + 2 = \frac{11}{2} \Rightarrow \alpha = \frac{7}{4}$$

$$\frac{c}{2} = \frac{7}{4}\left(\frac{7}{4} + 2\right) \Rightarrow c = \frac{7}{2}\left(\frac{15}{4}\right) = \frac{105}{8}$$

A

10)

D

11)

$2^x = 3$ or $2^x = 5$

$$\sum x = \log_2 15 = \frac{\log_{10} 15}{\log_{10} 2}$$

E

12)

Surface area $6xd + 4x^2 \times \frac{\sqrt{3}}{2} = 6xd + 2\sqrt{3}\,x^2$

Volume= $\sqrt{3}x^2d$

$$6xd + 2\sqrt{3}\,x^2 = \sqrt{3}\,x^2d$$

$$d = \frac{2\sqrt{3}x^2}{\sqrt{3}\,x^2 - 6x} = \frac{2x}{x - 2\sqrt{3}}$$

D

13)

Let $f(x) = x^4 - 4x^3 + 4x^2 - 10$

$$f'(x) = 4x^3 - 12x^2 + 8x$$

$$f'(x) = 0 \Rightarrow x = 0 \;\; or\; x^2 - 3x + 2 = 0$$

$x = 0, x = 1$ or $x = 2$

$$f(0) = -10$$

$$f(1) = -9$$

$$f(2) = -10$$

All turning points of $y = f(x)$ lie below the x axis.

There are two real roots.

C

14)

Straight line $\Rightarrow \log y = m \log x + c \Rightarrow \log y = \log x^m + \log c' \Rightarrow y = c'x^m$

With $c' = a$ and $m = b$ this is $y = ax^b$

D

15)

Minimum, when $a = \frac{1}{2}, = \int_{-1/2}^{1/2} x^2 dx = \frac{2\left(\frac{1}{2}\right)^3}{3} = \frac{1}{12}$

A

16)

$$\frac{10^{c-2d} \times 20^{2c+d}}{8^c \times 125^{c+d}} = \frac{2^{c-2d+4c+2d} \times 5^{c-2d+2c+d}}{2^{3c} \times 5^{3c+3d}} = \frac{2^{5c} \times 5^{3c-d}}{2^{3c} \times 5^{3c+3d}}$$

This is an integer if c and d are integers and $c > 0$ and $d < 0$.

E

17)

$$(a-2)^2 + 8a > 0$$

$$a^2 + 4a + 4 > 0$$

$$(a+2)^2 > 0$$

$$a \neq -2$$

D

18)

$$\sin 2x \geq \frac{1}{2} \text{ for } \frac{\pi}{6} \leq 2x \leq \frac{5\pi}{6}$$

$$\text{so } \frac{\pi}{12} \leq x \leq \frac{5\pi}{12}$$

$$-1 \leq \tan x \leq 1 \text{ for } 0 \leq x \leq \frac{\pi}{4} \qquad \left(\text{and for } \frac{3\pi}{4} \leq x \leq \pi\right)$$

$$\frac{\pi}{12} \leq x \leq \frac{\pi}{4}$$

The length of the interval is $\frac{\pi}{6}$

B

19)

$$4r - 4 = 4r^3 - 4r$$

$$r^3 - 2r + 1 = 0$$

$$(r-1)(r^2 + r - 1) = 0$$

$$r = \frac{-1+\sqrt{5}}{2}$$

$$S_\infty = \frac{4}{1 - \left(\frac{-1+\sqrt{5}}{2}\right)} = \frac{8}{3-\sqrt{5}} = \frac{24+8\sqrt{5}}{4} = 6 + 2\sqrt{5}$$

D

20)

The coefficient of x^2 in the expansion is the same as the coefficient of x^2 in the expansion of

$$4(1 + 2x + 3x^2)^6 \text{ which is } 4 \times \left(\binom{6}{2} \times 2^2 + 6 \times 3\right) = 4 \times 78 = 312$$

G