OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

ADDITIONAL MATHEMATICS **6993**

Summer 2003

Friday **13 JUNE 2003** Morning 2 hours

Additional materials:
Answer booklet
Graph paper

TIME 2 hours

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given correct to three significant figures where appropriate.
- The total number of marks for this paper is 100.

This question paper consists of 5 printed pages and 3 blank pages.

Section A

1 Solve simultaneously the equations $y = x + 6$ and $y = x^2 - x + 3$. [4]

2 (i) Show that there is a stationary point at $(1, 9)$ on the curve $y = x^3 - 6x^2 + 9x + 5$ and determine the nature of this stationary point. [5]

(ii) Find the coordinates of the other stationary point and hence sketch the curve. [2]

3 The gradient function of a curve is given by $\frac{dy}{dx} = 2 + 2x - x^2$. Find the equation of the curve given that it passes through the point (3, 10). [4]

4 Find the four values of θ in the range $0° \leqslant \theta \leqslant 360°$ that satisfy the equation $\sin 2\theta = 0.5$. [4]

5 (i) By drawing suitable graphs on the same axes, indicate the region for which the following inequalities hold. You should shade the region which is **not** required.

$$3x + 4y \leqslant 24$$
$$3x + y \leqslant 15$$
$$x \geqslant 0$$
$$y \geqslant 0$$

[5]

(ii) Find the maximum value of $2x + y$ subject to these conditions. [2]

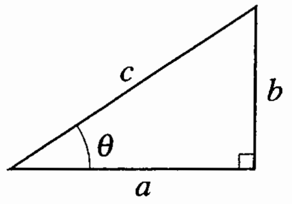
6 (i) Expand $(2 + x)^7$ in ascending powers of x up to and including the term in x^3. [4]

(ii) Use your expansion with an appropriate value of x to find an approximate value of 1.99^7. Give your answer to 4 decimal places.

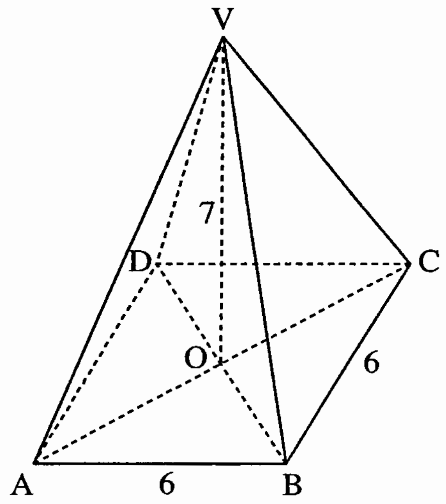
Show your working clearly, giving the numerical value of each term.

[Just writing down the value of 1.99^7 from your calculator will earn no marks.] [3]

7 Use the given triangle to prove that, for $0° < \theta < 90°$, $1 + \tan^2\theta = \dfrac{1}{\cos^2\theta}$. [3]



8 A pyramid ABCDV has a square, horizontal, base ABCD of side 6 cm. The vertex V is vertically above the centre of the base O. The pyramid has height 7 cm.



Find the angle that the sloping edge VA makes with the horizontal. [5]

9 The function $f(x)$ is defined by $f(x) = x^3 + 2x^2 - 5x - 6$.

(i) Show that when $f(x)$ is divided by $(x - 3)$ the remainder is 24. [2]

(ii) Show that $(x - 2)$ is a factor of $f(x)$. [1]

(iii) Hence solve the equation $f(x) = 0$. [4]

10 A car, which is initially travelling at 20 m s^{-1}, accelerates uniformly at 1.2 m s^{-2}.

Find

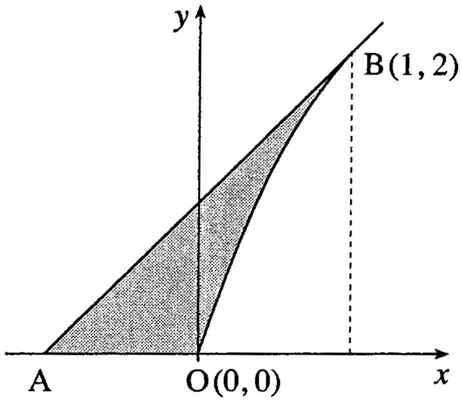
(i) the speed after 5 seconds, [2]

(ii) the distance travelled in this time. [2]

[Turn over

Section B

11 The shaded region on the diagram shows a boat's sail. The units are metres and, referred to the axes shown, the coordinates of O and B are $(0, 0)$ and $(1, 2)$ respectively. OB is part of the curve $y = 3x - x^2$. The tangent to the curve at B meets the x-axis at A.



Find

(i) $\frac{dy}{dx}$ for the curve $y = 3x - x^2$, [2]

(ii) the equation of the tangent at B, [3]

(iii) the coordinates of the point A, [1]

(iv) the area of the sail. [6]

12 **(a)** A quality control officer inspects a large batch of electric light bulbs which are in packs of 8. He chooses a pack at random and tests all the bulbs to see if they are working. On this day 10% of the bulbs are faulty.

Find the probability that in the pack chosen

(i) none is faulty, [2]

(ii) two or more are faulty. [5]

(b) If the officer finds no faulty bulbs, he accepts the whole batch. If he finds two or more faulty bulbs he rejects the whole batch. If he finds one faulty bulb then he chooses a second pack at random and accepts the whole batch only if this second pack has no faulty bulbs.

Find the probability that the whole batch is accepted. [5]

13 There are 10 tonnes of potatoes in a large container. Bags of potatoes of nominal mass 5 kg are filled from this container. The potatoes are not all the same size and it is not possible to make the bags exactly 5 kg. [1 tonne = 1000 kg.]

(i) If all bags could be made with a mass of exactly 5 kg, how many bags would be filled from the container? [1]

The bags could be too light by up to x kg or too heavy by up to x kg.

(ii) State, in terms of x, the largest and smallest number of bags that can be filled from the container. [2]

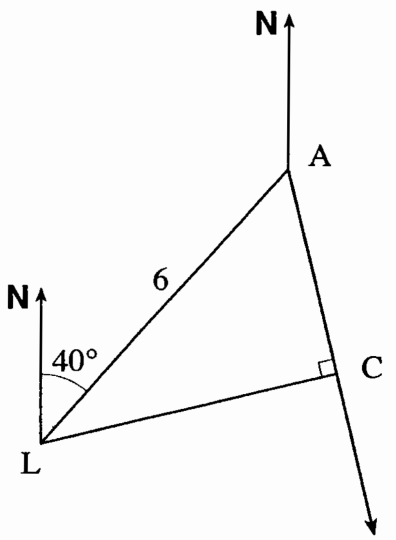
(iii) Given that the largest number of bags is 100 more than the smallest number of bags, write down an equation in x and show that it simplifies to $x^2 + 200x - 25 = 0$. [6]

(iv) Solve this equation and hence work out the largest and smallest mass of a bag of potatoes. [3]

14 At 1200 a ship is at a point A on a bearing of 040° from a lighthouse L and at a distance of 6 nautical miles.

The ship is moving on a bearing of 170° at 21 knots. [1 knot is a speed of 1 nautical mile per hour.]

C is the point where the ship is nearest to the lighthouse.



Not to scale

(i) Show that the angle LAC = 50°. [1]

(ii) Find the distance LC and the time when the ship is at the point C. [6]

(iii) At what time is the ship on a bearing of 110° from the lighthouse? [5]

Mark Scheme

OCR
RECOGNISING ACHIEVEMENT

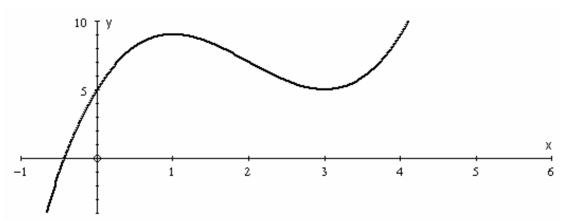
FSMQ Additional Mathematics

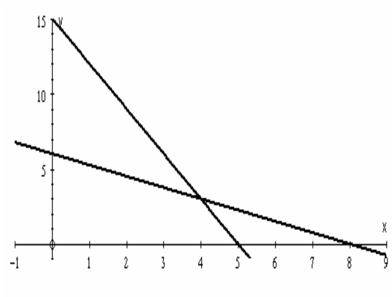
6993

Mark Scheme, June 2003

6993 Additional Mathematics Summer 2003 Solutions

Section A

1	$x+6=x^2-x+3 \Rightarrow x^2-2x-3=0$ $\Rightarrow (x-3)(x+1)=0 \Rightarrow (3, 9),\ (-1, 5)$ Alternatively: eliminate y giving $y^2 - 14y + 45 = 0$	**4**	M1 A1 A1 A1	Or A1 for 3, -1 A1 for 9, 5
2	(i) $\frac{dy}{dx}=3x^2-12x+9=0$ when $x^2-4x+3=0$ $\Rightarrow (x-3)(x-1)=0 \Rightarrow x=1 \Rightarrow y=9$ $\frac{d^2y}{dx^2}=6x-12<0$ when $x=1 \Rightarrow$ maximum (Alternatively B1 B1 for +ve on left and -ve on right of stationary point.)	**5**	M1 Diff A1 getting 0 B1 getting 9 DM1 A1	Dep. On 1st M1
	(ii) Other stationary point is (3, 5) 	**2** **7**	B1 for other stationary point F1 for sketch (General shape)	Conforms to 2nd stationary point.
3	$\frac{dy}{dx}=2+2x-x^2 \Rightarrow y=2x+x^2-\frac{x^3}{3}\ \ (+c)$ Through (3, 10) $\Rightarrow 6+9-9+c=10 \Rightarrow c=4$ $\Rightarrow y=2x+x^2-\frac{x^3}{3}+4$	**4**	M1 (ignore c) A1 all correct M1 for c F1	Increase in powers evident
4	$\sin 2\theta=0.5 \Rightarrow 2\theta=30$ and 150 and also $2\theta=390$ and also $2\theta=510$ $\Rightarrow \theta=15,\ 75,\ 195,\ 255$	**4**	M1 Solving A1 for 15 F2,1 remaining 3, -1 each error	

5	 (i) $3x+4y=24$ $3x+\ \ y=15$ meet at (4, 3) (ii) Max value of $2x+y$ is 11	5 2 **7**	B1 B1 each line B1 B1 each shading B1 for (4,3) M1 F1 for 11	Ignore $x \geq 0$, $y \geq 0$ Award this where used or seen
6	(i) $(2+x)^7 = 2^7 + 7.2^6x + 21.2^5x^2 + 35.2^4x^3 + ...$ $= 128 + 448x + 672x^2 + 560x^3 + ...$ (ii) Substitute $x = -0.01$ $\Rightarrow 1.99^7 = 128 - 4.48 + 0.0672 - 0.000560 + ...$ $= 123.5866$	4 3 **7**	B1 powers (x and 2s) B1 coefficients B2 all terms, (B1 1 error) M1 substitute F1 terms A1 final answer	 0.01 or -0.01
7	L.H.S. = 1 + $\tan^2\theta = 1 + \frac{b^2}{a^2} = \frac{a^2+b^2}{a^2} = \frac{c^2}{a^2} = \frac{1}{\cos^2\theta}$ =R.H.S	**3**	B1 B1 B1	Starting with end equation B2 only
8	$AB = 6 \Rightarrow AC = 6\sqrt{2} \approx 8.485 \Rightarrow AO = 3\sqrt{2} \approx 4.243$ $\Rightarrow \tan\theta = \frac{7}{3\sqrt{2}} \approx 1.650 \Rightarrow \theta = 58.8^0$	**5**	M1 A1 B1 Correct angle M1 A1	Application of Pythagoras in ABC Must be angle in correct triangle
9	(i) $f(x) = x^3 + 2x^2 - 5x - 6 \Rightarrow f(3) = 27 + 18 - 15 - 6 = 24$ (ii) f(2) = 8 + 8 −10 − 6 = 0 (iii) f(x) = $(x-2)(x^2+4x+3) = (x-2)(x+1)(x+3)$ $\Rightarrow$ f(x)=0 $\Rightarrow (x-2)(x+1)(x+3) = 0 \Rightarrow x = -3, -1, 2$	2 1 4 **7**	M1 A1 B1 M1 A1 A1 A1	 For each linear term. Or use of factor theorem.
10	(i) $v = u + at \Rightarrow v = 20 + 1.2 \times 5 = 26\text{ms}^{-1}$ (ii) $s = ut + \frac{1}{2}at^2 \Rightarrow s = 100 + 0.6 \times 25 = 115\text{m}$	2 2 **4**	M1 A1 M1 A1	-1 max penalty for incorrect or missing units

Section B

11	(i) $y=3x-x^2 \Rightarrow \frac{dy}{dx}=3-2x.$	**2**	M1 A1	
	(ii) When $x=1, g=1 \Rightarrow y-2=1(x-1) \Rightarrow y=x+1$	**3**	F1 M1 A1	
	(iii) When $y=0,\ x=-1 \Rightarrow$ A(-1,0)	**1**	F1	
	(iv) Area under curve = $\int_0^1 (3x-x^2)\,dx$		M1	Must have correct limits
	$=\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^1=\frac{3}{2}-\frac{1}{3}=\frac{7}{6}\ \text{m}^2$		A1 A1	
	Area triangle = $\frac{1}{2}.2.2=2\ \ \text{m}^2$		F1	
	Total Area = 2 − $\frac{7}{6}=\frac{5}{6}\ \ \text{m}^2$	**6**	M1 F1	
		12		
12	**(a)** **(i)** $\left(\frac{9}{10}\right)^8=0.430$	**2**	M1 A1	
	(ii) $1-\left(\frac{9}{10}\right)^8-8.\left(\frac{9}{10}\right)^7\left(\frac{1}{10}\right)$ $=1-0.4305-0.3826=0.187$	**5**	M1 (1 -) A1 (2 terms) B1 (powers) B1 (coeff) A1 (ans)	Alt: M1 other terms A1 7 terms B1 powers B1 coeffs
	(b) (a)(i) + (a)(i) × P(1 faulty) = 0.4305 + 0.4305 × 0.3826 = 0.595	**5**	F1 (1st term) M1 F1 (2nd term) M1(add) A1 (ans)	Their (a)(i) Mult of 2 terms 3 or 4 sig figs
		12		

13	(i) $\frac{10000}{5} = 2000$	**1**	B1	Special case: Allow with 1000 and follow through. Accept either order
	(ii) $\frac{10000}{5-x}$, $\frac{10000}{5+x}$	**2**	B1 B1	
	(iii) $\frac{10000}{5-x} - \frac{10000}{5+x} = 100$		M1 A1	
	$\Rightarrow 10000\left((5+x)-(5-x)\right) = 100\left(25-x^2\right)$		M1 A2,1	(even if wrong way round)
	$\Rightarrow x^2 + 200x - 25 = 0$	**6**	A1	Clearing fractions
	(iv) $\Rightarrow x = \frac{-200 \pm \sqrt{200^2 + 100}}{2} \approx 0.125$		M1 A1	
	$\Rightarrow$ largest = 5.125, smallest 4.875	**3**	A1	
		12		
14	(i) Angle LAC = 40 + 10 = 50	**1**	B1	
	(ii) LC = 6sin50 = 4.596		M1 A1	
	AC = 6cos50 = 3.857		A1	
	$\Rightarrow$ Time = $\frac{3.857}{21} = 0.1837\text{hrs} \approx 11\text{mins}$		M1 A1	
	$\Rightarrow 1211$	**6**	F1	
	(iii) $\frac{AB}{\sin 70} = \frac{6}{\sin(180-70-50)}$		M1 A1	
	$\Rightarrow AB = 6 \times \frac{\sin 70}{\sin 60} \approx 6.510$		A1	
	$\Rightarrow$ Time = $\frac{6.510}{21} = 0.31\text{hrs} \approx 18.6\text{mins}$	**5**	M1 F1 (allow 19 mins)	
	$\Rightarrow 1219$			
	Alt. (iii)			
	Bearing = 110^0, Angle ALC = 40^0.			M1
	If position is B then angle CLB = 30^0			A1 (for 30)
	CB = LCTan30 = 2.65			A1
	Time = $\frac{2.65}{21} \times 60 = 8\,\text{mins}$			M1
	1211 + 8 = 1219	**12**		F1

Examiner's Report

Report on 6993. Additional Mathematics Summer 2003.

This is the first year of this specification. It replaced the old Additional Mathematics syllabuses, 8645 (the old O&C "traditional paper" and 8647, the MEI Additional Mathematics syllabus) and was geared to the same sort of candidature. The regulations for an Advanced FSMQ, as stated clearly in the specification, is that the appropriate starting point is a good grade at Higher Tier. Many of the candidates had clearly started from here, and had found that the course did not challenge them in quite the same way. There were many high scores including some full marks which was most pleasing to see. However, it was equally clear that many candidates had not started from the appropriate place. Many candidates not only failed to demonstrate any understanding of the extension material but failed to demonstrate the sort of understanding of some Higher Tier topics. Centres are encouraged to seek advice, if necessary, to find the most appropriate course for their students.
Because it is the first year of this specification the comments below are rather more full than they might be; it is hoped that teachers will use this report to inform them of requirements and standards expected.

1 This is a typical case in point. The question required candidates top find the intersection of a curve and a line and this is a standard Higher Tier topic. As such it should have been a relatively easy start to the paper. Yet many failed to show any understanding of the basic algebraic skills required to make the substitution in order to obtain the quadratic to be solved.
Some substituted for x, giving a quadratic in y. While this clearly results in the correct answer, it is rather more complicated to do. Many candidates failed to find the values of the other variable. Solving simultaneously means find the pairs (x, y) and is the same question as "find the coordinates of the intersection of the curve and the line".

[(-1, 5), (3, 9)]

2 Many candidates failed to show that the stationary point was at (1, 9), demonstrating simply that there was a turning point when $x = 1$. The "nature" of the turning point was done by either the second derivative, the "direction" of the gradient around the turning point or, having established $y = 9$ at the turning point to find values of y close either side of $x = 1$. This last method requires the verification of $y = 9$ when $x = 1$ which, as stated above, was often omitted.
The "sketch" was often rather too sloppy for the mark. Candidates are expected to show intercepts on the axes and the turning points for their mark.

[(ii) (3, 5)]

3 A simple integration question which was well answered; only a small minority of candidates failed to calculate the constant of integration. A typical indication of an inappropriate entry was the candidate who took the "m" of $y = mx + c$ to be $2 + 2x - x^2$, writing the equation as
$y = (2 + 2x - x^2)x + c$.

[$y = 2x + x^2 - \frac{x^3}{3} + 4$]

4 Many students managed $2\theta = 30 \Rightarrow \theta = 15$, but failed to find the other roots.The information that there were four roots should have given a clue but a number of candidates ignored it!
Rather more worrying were the candidates who wrote $\sin 2\theta = 0.5 \Rightarrow \sin\theta = 0.25 \Rightarrow \theta = 14.5^0$ or even worse, $\sin 2\theta = 0.5 \Rightarrow \theta = \dfrac{0.5}{\sin 2} = \dfrac{0.5}{0.0349} = 14.32^0$

[15^0, 75^0, 195^0, 255^0]

5 The first part of this question was, in general, well done, although some candidates were unable to draw the lines properly. The maximum value of $2x + y$ was not so well done; the good candidates realised that it occurred at the intersection while others found the value at all corners in order to deduce. For this short question in Section A it was not a requirement to justify that the point of intersection was indeed (4, 3) or that this was the point that maximised the objective function.

[(ii) 11 at (4, 3)]

6 The binomial expansion had not been covered by many; some tried to multiply it all out while others wrote incorrect algebra. When a numeric value is to be found "without use of the calculator" the values of all terms was expected to be seen. While no candidate simply wrote down the answer (which would have come from the calculator) many wrote down incorrect arithmetic (for instance all terms positive) and then the right answer. Candidates who are clever enough to take short cuts should remember to write down everything in situations like this.

[(i) $128 + 448x + 672x^2 + 560x^3 + \ldots\ldots$ (ii) 123.5866]

7 This question was an A grade discriminator!! Those candidates who were able to do the question usually started with the equality given, manipulating both sides to give something that they knew to be true (i.e. Pythagoras) rather than start with one side of the identity and to manipulate to give the other side.

8 There were some surprising answers here! Many candidates got it right but by a very roundabout way. The intention was to find AO and then in the triangle VOA use the tan ratio to give the answer. Many did not do this. A common alternative seen was to find the hypotenuse by Pythagoras and then use the sin or cos ratio, thus involving an extra step. Another was to find AV and CV (deduced correctly to be equal) and then to apply the cosine formula on triangle AVC.

[58.8^0]

9 This is a case where all working needs to be seen, even though it is very simple for the able candidate. Examiners need to see the substitution of $x = 3$ into the function to yield 24.
Some did part (iii) by attempting to divide by $(x – 2)$ while others found the other roots by trail and error. Examiners did not report seeing any candidate using the logic of trying only certain numbers due to the fact that the produce of the three roots had to be 6.
Some lost the last mark by factorising f(x) only without solving the equation f(x) = 0.

[(iii) x = 2, -1, -3]

10 Part (i) was often logically deduced without any clear statement of one of the standard formulae, but (ii) caused extra problems. Some effectively used $s = \frac{(u+v)}{2}t$ for each second to achieve the right answer. There was not much evidence that these formulae were well known. We expected to see the units given in these answers.

[(i) 26 ms^{-1}, (ii) 115m]

11 This question was often well done. The weaker candidates were able to differentiate the function to find the gradient function but then failed to obtain the equation of the tangent. The weakest, of course, were unable even to differentiate.

Because of the limits of each area the method of using $\int_a^b (y_1 - y_2)\,\mathrm{d}x$ was clearly not appropriate.

We were surprised that so many integrated the equation of the tangent to finds the are under the line instead of seeing it as a triangle.

$\left[\text{(i)} \ \frac{\mathrm{d}y}{\mathrm{d}x} = 3 - 2x. \quad \text{(ii)} \ y = x + 1 \ \ \text{(iii)} \ (-1,0) \quad \text{(iv)} \ 2 - \frac{7}{6} = \frac{5}{6} \ \text{m}^2 \right]$

12 This was a standard binomial distribution question set in previous Additional Mathematics papers and many completed it well and with the minimum of fuss (though some lost a mark through not correcting answers to 3 significant figures as required by the rubric – some gave far too many and others approximated prematurely). The same proportion of candidates completing the question failed to involve the binomial coefficients. Many weak candidates misread the question and worked on $\left(\frac{1}{8}\right)^{10}$ or similar, incorrect, terms.

[(i) 0.430, (ii) 0.187 (iii) 0.595]

13 Unfortunately many candidates did not know the connection between tonnes and kilograms though compensatory marks were available. In spite of this many were able to set up the equation $\frac{10000}{5-x} - \frac{10000}{5+x} = 100$. The main problem arose over the algebraic manipulation resulting in the quadratic equation $x^2 + 230x - 25 = 0$. Needless to say, incorrect algebra leading to the correct answer resulted in no marks! Candidates need to be aware that examiners cannot be fooled, for when the answer is given we look very carefully at all the steps that lead to it.
Some were able to re-enter the question and simply solve the quadratic equation.

[(i)2000, (ii) $\frac{10000}{5-x}, \frac{10000}{5+x}$, (iv) 5.125 kg, 4.875 kg]

14 Justification of the angle of 50^0 was often not done and lengths were sometimes corrected too soon. Unfortunately, some candidates did not read the question properly and, having worked out LC as required, then used that length to work out the “time” rather than understand that, although it was not asked for, it was the length AC that was required. Two methods were seen for the last part; some worked out the “extra” bit, getting an angle of 30^0 in a right-angled triangle and adding the time on, while others started from the beginning again with a triangle that required the use of the sine rule.

[(ii) LC = 4.596 km, time = 1211 (iii) 1219]

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ADDITIONAL MATHEMATICS **6993**

Summer 2004

Monday **21 JUNE 2004** Morning 2 hours

Additional materials:
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[Turn over

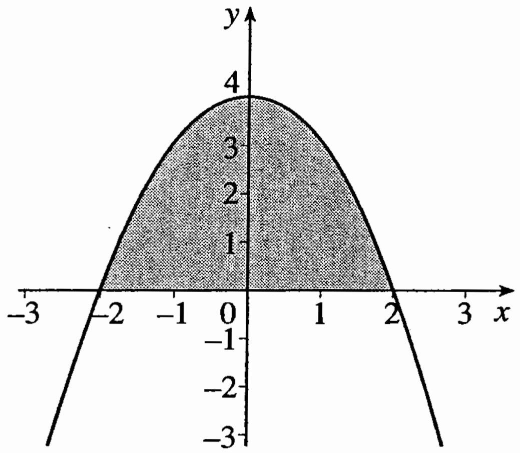
Section A

1 The vertices of a quadrilateral are $A(-2, 0)$, $B(2, 2)$, $C(7, -3)$ and $D(0, -4)$.

(i) Calculate the gradients of the diagonals AC and BD and state a geometrical fact about these lines. [3]

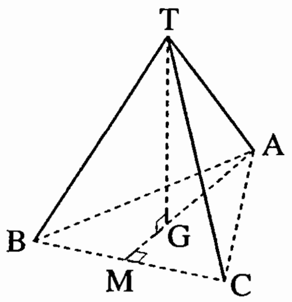
(ii) Show that the mid-point of BD lies on AC. [3]

2 The curve shown is part of the graph of $y = 4 - x^2$.



Calculate the area of the shaded region between this curve and the x-axis, giving your answer as an exact fraction. [4]

3 A tripod with vertex T stands on level ground. The three legs TA, TB and TC are each 60 cm long. The triangle ABC is equilateral with side 50 cm. The point M is the mid-point of BC, and G lies on MA such that MG : GA = 1 : 2. You are given that T lies vertically above G.



Find the angle which the leg TA makes with the ground. [5]

4 Find, by calculus methods, the x-coordinate of the minimum point on the curve

$$y = 2x^3 - 3x^2 - 12x + 6.$$

Show your working clearly, giving the reasons for your answer. [5]

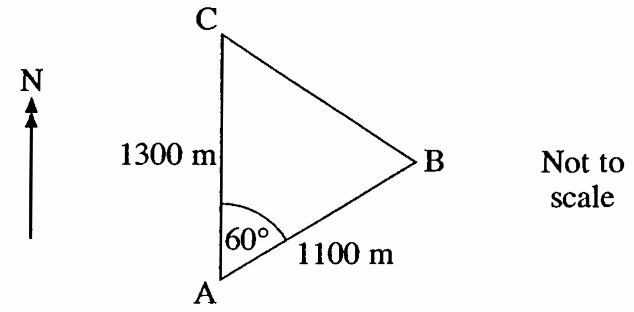
5 (a) Find the value of x in the range $0° < x < 360°$ that satisfies **both** $\tan x = 0.75$ **and** $\cos x = -0.8$. [3]

(b) Find all the values of x in the range $0° < x < 360°$ that satisfy $\sin x = -2\cos x$. [3]

6 Express $(2 + \sqrt{3})^3$ in the form $a + b\sqrt{3}$ where a and b are integers. [4]

7 Find the points of intersection of the line $y = 3x + 1$ with the circle $x^2 + y^2 = 12$, giving your answers correct to 2 decimal places. [5]

8 The points A, B, C are three points on an orienteering course. B is 1100 metres from A on a bearing of 060°. C is 1300 metres from A and due north from A.



(i) Show that BC = 1212 metres, correct to the nearest metre. [2]

(ii) Hence find the bearing of C from B. [3]

9 The probability that I am late for work on any given day is 0.1. Being late on one day is independent of any other day.

Find the probability that in a week of 5 working days I am late at least twice. Give your answer correct to 4 decimal places. [5]

[Turn over

10 **(i)** Express $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$ where a and b are integers. [3]

(ii) Hence show that the equation $f(x) = 0$ has no real roots. [1]

You are given that $g(x) = x^3 + 4x^2 - x - 22$.

(iii) Show that $g(2) = 0$. [1]

(iv) Hence show that the equation $g(x) = 0$ has only one real root. [2]

Section B

11 A taxi firm plans to change its fleet of vehicles by buying MPVs (multi-purpose vehicles) and cars. MPVs can carry up to 7 passengers, and cars can carry up to 4 passengers.

MPVs cost £20 000 and cars cost £9000.

The firm can spend up to £180 000.

There is a maximum of 12 drivers available at any one time.

Denoting the number of MPVs to be bought as x and the number of cars to be bought as y, form two inequalities in x and y. [3]

On a graph draw suitable lines and shade the region that the inequalities do **not** allow. [Take 1 cm to represent 1 vehicle.] [4]

From your graph find a pair of values (x, y)

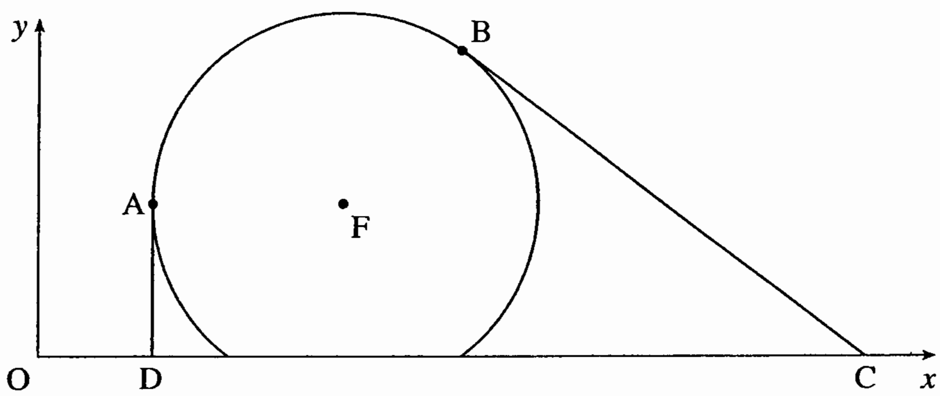
(i) which uses all the £180 000, [1]

(ii) which uses all 12 drivers but minimises the expenditure on vehicles, [1]

(iii) which maximises the number of people that can be carried simultaneously. [3]

12 The shape shown in the diagram is part of a circle. The centre of the circle is F(8, 4) and AD and BC are tangents at A and B respectively. A is the point (3, 4) and B is the point (11, 8).

A wire is stretched from D to A, round the circumference of the circle to B and then to C, where D and C are on the x-axis. Units are centimetres.



(a) Find the equation of the circle. [3]

(b) **(i)** Find the gradient of FB and hence the equation of the tangent BC. [4]

(ii) Given that the length of the wire from A to B in contact with the circle is 11.07 cm, correct to 2 decimal places, find the total length of the wire. [5]

[Turn over

13 I regularly travel a journey of 200 kilometres. When I travel by day I average v kilometres per hour. When I travel at night the traffic is not so bad, so I can average 20 kilometres per hour faster. This means that I am able to complete the journey in 50 minutes less time.

(i) Write down expressions for the journey times during the day and at night. [2]

(ii) Hence form an equation in v and show that it simplifies to

$$v^2 + 20v - 4800 = 0.$$ [5]

(iii) Hence find the times it takes me to complete the journey during the day and at night. [5]

14 A car starts from rest and reaches $20\,\text{m}\,\text{s}^{-1}$ in 8 seconds.

(a) Jane models the motion of the car by assuming constant acceleration during the first 8 seconds.

(i) Find the value of the constant acceleration. [2]

(ii) Find the distance travelled during this time. [2]

(b) Paul claims that constant acceleration is not a good model in this situation.

He uses the following formula for the velocity, $v\,\text{m}\,\text{s}^{-1}$, at time t seconds for the first 8 seconds of motion.

$$v = \frac{60t^2 - 5t^3}{64}$$

(i) Show that this formula does give $v = 0$ when $t = 0$ and $v = 20$ when $t = 8$. [1]

(ii) Find the acceleration when $t = 8$. [3]

(iii) Find the distance travelled during the first 8 seconds using this model. [4]

Mark Scheme

RECOGNISING ACHIEVEMENT

June 2004

ADVANCED FSMQ

MARK SCHEME

Maximum mark: 100

Syllabus/component:

6993 Additional Mathematics

Paper set Date: June 21, 2004

Mark scheme

6993 Additional Mathematics. June 2004 Mark scheme

1	**(i)**	Gradient AC = $\frac{-3-0}{7--2}=\frac{-3}{9}=-\frac{1}{3}$ Gradient BD = $\frac{-4-2}{0-2}=\frac{-6}{-2}=3$ Since $3\times-\frac{1}{3}=-1$ the lines are perpendicular	B1 B1 B1 **3**	
	(ii)	Mid - point of BD = $\left(\frac{2+0}{2},\frac{2+-4}{2}\right)=(1,-1)$ Grad AM $=\frac{-1-0}{1--2}=-\frac{1}{3}=$ Grad AC $\Rightarrow$ points collinear Alternatively: Equation of AC is $x+3y+2=0$ This equation is satisfied by (1, -1) as $1-3+2=0$	B1 B1 B 1 **3**	B1 B1
2		Area = $\int_{-2}^{2}(4-x^2)\,dx=\left[4x-\frac{x^3}{3}\right]_{-2}^{2}$ $=\left(8-\frac{8}{3}\right)-\left(-8-\frac{-8}{3}\right)=16-\frac{16}{3}=\frac{32}{3}$	M1 A1 M1 A1 **4**	Integrate or $2\times\int_0^2$ Substitute
3		M is the mid-point of BC ⇒BM = 25 cm. By Pythagoras, MA = $\sqrt{50^2-25^2}=25\sqrt{3}=43.3$ $\Rightarrow$ AG $=\frac{50}{3}\sqrt{3}=28.87$ cm $\Rightarrow$ angle $=\cos^{-1}\frac{28.87}{60}=61.2^0$ **Alt:** Find angle by cosine rule of TAM Find AM and TM M1 AM correct A1 TM correct A1 Cosine rule used M1 A1	M1 A1 F1 M1 A1 **5**	For MA For AG
4		$\frac{dy}{dx}=6x^2-6x-12$ $=0$ when $x^2-x-2=0$ $\Rightarrow(x-2)(x+1)=0\Rightarrow x=2,-1$ By considering sign of grad either side of turning point $\Rightarrow$ Minimum at $x=2$ Alternatively: $\frac{d^2y}{dx^2}=12x-6$: When $x=2$ $\frac{d^2y}{dx^2}>0\Rightarrow$ Minimum	B1 M1 A1 M1 A1 **5**	M1 A1

5	**(a)**	From calculator, $\tan^{-1}0.75 = 36.9$ There is a second angle in the third quadrant, where the cos value is negative i.e. $x = 180 + 36.9 = 216.9^0$	B1 B1 B1 **3**	 Only 1 answer. Accept anything that is correct.
	(b)	$\tan x = -2$ $\Rightarrow 116.6^0$ and 296.6^0 Alt: square both sides $\Rightarrow \sin^2 x = 4\cos^2 x \Rightarrow \sin^2 x = 4 - 4\sin^2 x$ $\Rightarrow 5\sin^2 x = 4 \Rightarrow \sin x = \pm\frac{2}{\sqrt{5}} \Rightarrow x = 63.4$ Sorting out correct quadrants to give correct angles N.B. any extra angles means no "sorting" of quadrants So M0	B1 B1 B1 **3**	For Tan*x* -1 extra values in range B1 Pythagoras B1 63.4 B1 two answers
6		$\left(2+\sqrt{3}\right)^3 = 2^3 + 3.2^2\sqrt{3} + 3.2.3 + 3\sqrt{3}$ $= 8 + 12\sqrt{3} + 18 + 3\sqrt{3}$ $= 26 + 15\sqrt{3}$ **Alt**. Multiply out 3 brackets then mult by third M1 For 7 A1 for $4\sqrt{3}$ A1 Answer A1 **Alt**: Mult everything by everything else (i.e. pick out 8 numbers) M1 4 terms correct A1 Other 4 terms correct A1 Collect up A1	M1 A2 A1 **4**	Binomial, includes coefficients all terms A1 one error collection of integers and surds
7		Substitute for *y*: $x^2 + (3x+1)^2 = 12 \Rightarrow 10x^2 + 6x - 11 = 0$ $\Rightarrow x = \frac{-6 \pm \sqrt{36+440}}{20} = \frac{-6 \pm \sqrt{476}}{20} = \frac{-6 \pm 21.82}{20}$ $= -1.39$ or $0.79 \Rightarrow y =$ -3.17 or 3.37 i.e. (-1.39, -3.17) or (0.79, 3.37) **Alt:** Sub for *x* to give quadratic in *y*: $10y^2 - 2y - 107 = 0$	M1 A1 M1 A1 A1 **5**	Get quadratic Use formula **Alt**: Trial and improvement to 2 dp Both *x* Both *y* Alt. A1 one pair, A1 the other pair

8	**(i)**	Cosine rule gives $BC^2 = 1100^2 + 1300^2 - 2.1100.1300.\cos 60 = 147000$ $\Rightarrow$ BC = 1212 metres	M1 A1 **2**	
	(ii)	sin rule gives $\frac{\sin C}{1100} = \frac{\sin 60}{1212} \Rightarrow C = 51.8$ $\Rightarrow$ Bearing $= 360 - 51.8 = 308^0$ Or 308.2 or 308.3 (*Do not accept any more decimal places*)	M1 A1 F1 **3**	Or B = 68.2 or 68.3 Alternative methods include cosine rule and splitting triangle into two right angled triangles
9		$P(0) = 0.9^5 = 0.59049$; $P(1) = 5.(0.9)^4.(0.1) = 0.3281$ $P(\text{ at least twice}) = 1 - P(0) - P(1)$ $= 1 - 0.5905 - 0.3281 = 0.0815$ Alt: Add 4 terms: B3,2,1 for the terms. -1 each error or omission. M1 Add the 4 terms. A1 ans Special case P(2) = 0.0729 B2	B1 B2 M1 A1 **5**	Including coefficient
10	**(i)**	$(x + 3)^2 = x^2 + 6x + 9$. So $x^2 + 6x + 11 = (x + 3)^2 + 2$	B1 B1 **2**	For 3, 2
	(ii)	$f(x) = 0 \Rightarrow (x + 3)^2 = -2$ which is never true.	B1 **1**	
	(iii)	$g(2) = 8 + 16 - 2 - 22 = 0$	B1 **1**	
	(iv)	$g(x) = (x - 2)(x^2 + 6x + 11)$ $\Rightarrow g(x) = 0 \Rightarrow (x - 2)(x^2 + 6x + 11) = 0$ $\Rightarrow x = 2$ as quadratic has no roots	M1 A1 A1 **3**	A comment must be made about the quadratic

Section B

11		Cost: 20 000x + 9 000$y \le$ 180 000 gives 20x + 9$y \le$ 180. Drivers: $x + y \le 12$ Graphs [These meet at (6.54, 5.45).] N.B. If inequalities not given but lines and shading correct on graph then give M1 for implied inequalities. If written along lines then give full marks	M1 A1 B1 B4,3,2,1 **7**	 B1 axes (whole area on page, axes labelled x,y or by definition B1 B1 each line B1 correct shading
	(i)	(9, 0)	B1 **1**	
	(ii)	(0, 12)	B1 **1**	
	(iii)	The value of $P = 7x + 4y$ at this point is 67.65 So the maximum value of P is 66 or 67 $7x + 4y = 67$ does not pass through any point $7x + 4y = 66$ passes through the point (6, 6) . Alt: Try "near points", e.g. (6, 6), (7, 4) Giving 66 at (6, 6)	M1 M1 A1 **3**	For correct point For integer values required M1 M1 (integer points) A1
12	**(a)**	Radius = 5 $\Rightarrow (x-8)^2 + (y-4)^2 = 25$	B1 M1 A1 **3**	
	(b)(i)	For tangent at B: Grad FB = $^4/_3$ $\Rightarrow$ grad of tangent at B = $-^3/_4$ $\Rightarrow 3x + 4y = 65$	B1 M1 M1 A1 **4**	For gradient For gradient For equation
	(b)(ii)	AD = 4 Tangent at B meets x axis at $(21^2/_3, 0)$ $\Rightarrow \text{BC} = \sqrt{\left(21\frac{2}{3}-11\right)^2+(8-0)^2} = 13.33$ AB = 11.07 $\Rightarrow$ Total length = 28.4 cm	B1 E1 M1 A1 F1 **5**	 Depends on their equation in (b)(i) Correct application of pythagoras Depends on the M mark

13	**(i)**	Time during the day = $\frac{200}{v}$ Time during the night = $\frac{200}{v+20}$	B1 B1 **2**	
	(ii)	$\frac{200}{v}-\frac{200}{v+20}=\frac{50}{60}$ $\Rightarrow 200(v+20)-200v=\frac{5}{6}v(v+20)$ $\Rightarrow 4000\times 6=5v(v+20)$ $\Rightarrow 5v^2+100v=24000$ $\Rightarrow v^2+20v-4800=0$	M1 A1 M1 A1 A1 **5**	Units correct (i.e. divide by 60) Manipulate
	(iii)	$v=\frac{-20\pm\sqrt{20^2-4.(-4800)}}{2}=\frac{-20\pm\sqrt{19600}}{2}=\frac{-20\pm 140}{2}$ $v=-10\pm 70=60$ $\Rightarrow$ Day speed = 60 $\Rightarrow$ Day time = $\frac{200}{60}$ = 3 hrs 20 mins $\Rightarrow$ night speed = 80 $\Rightarrow$ night time = $\frac{200}{80}$ = 2 hrs 30 mins Alt: Night time = day time – 50 mins	M1 A1 A1 F1 F1 **5**	Correct formula For speed Alt: Factorise M1 Corr. brackets A1 For 60 A1 For each time (Allow 3 $^1/_3$ hrs)
14	**(a)(i)**	$v=at\Rightarrow a=\frac{20}{8}=2.5\text{ ms}^{-2}$	M1 A1 **2**	Units required in (a). -1 once if units wrong or missing.
	(a)(ii)	$s=\frac{1}{2}at^2\Rightarrow s=80\text{m}$	M1 A1 **2**	
	(b)(i)	Substitute $t=0$ to give $v=0$ and $t=8$ to give $v=20$	B1 **1**	
	(b)(ii)	Differentiate: $a=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{120t-15t^2}{64}$ When $t=8$ $a=0$ Special Case: If 64 left off, M1 A0 B1	M1 A1 B1 **3**	
	(b)(iii)	Integrate: $s=\int_0^8\frac{60t^2-5t^3}{64}\,\mathrm{d}t=\left[\frac{20t^3-\frac{5}{4}t^4}{64}\right]_0^8$ $=\left(\frac{10240-5120}{64}\right)=80\text{m}$	M1 A1 M1 A1 **4**	Integrate Substitute

Examiner's Report

FSMQ Additional Mathematics (6993) Report, Summer 2004

The regulations for an Advanced FSMQ, as stated clearly in the specification, is that the appropriate starting point is a good grade at Higher Tier. Many candidates started from here and achieved good scores, including some full marks, which was most encouraging. However, as stated last year, it was equally clear that many candidates had not started from the appropriate place. Many candidates not only failed to demonstrate any understanding of the extension material but failed to demonstrate the sort of understanding of some Higher Tier topics. There were a distressing number of candidates scoring very low marks, including single figures and 0. This cannot have been a positive experience for them. Centres are encouraged to seek advice, if necessary, to find the most appropriate course for their students and to seek for further advice on this particular course.
The mean mark was 50.4, down 8 marks from last year, indicating that candidates found the paper this year a little more difficult. The thresholds were reduced accordingly.

Section A
Q1. (Coordinate Geometry)
Weaker candidates usually struggle with this topic, but the basic focus of this question should have enabled all candidates to get started. In fact a number were not able to find the gradient of a line given two points on it. In part (ii) the mid-point was also often wrong. Most candidates found the mid-point, found the equation of AC and showed that one lay on the other. Very few appealed to geometry, or found gradients of, say, AM and AC showing them to be equal.
[Gradients 3 and $-\frac{1}{3}$; lines perpendicular.; Midpoint $(1, -1)$]

Q2. (Area under curve).
In spite of the diagram given not everyone used the correct limits, and then a number got the arithmetic wrong at the end, even offering an answer of 0!!
[Area $= 10\frac{2}{3}$]

Q3. (Trigonometry on 3-D shape)
Often well done, but the greatest error was a failure to use Pythagoras properly taking $AM^2 = 50^2 + 25^2$. A few used other methods, such as the cosine rule and a number did not take enough care over the information given, interchanging 50 and 60.
[61.2^0.]

Q4. (Stationary values).
Most candidates were able to differentiate. Not quite so many were able to deal with the common factor of 6 in the process of solving $\frac{dy}{dx} = 0$. Even fewer were able to justify mathematically the minimum point..
[$x = 2$.]

Q5. (Trigonometrical ratios for angles greater than 90^0.)
Candidates were not comfortable with this question. In part (i) the idea of one ratio being positive and another negative giving the quadrant within which the angle lay was not familiar with most candidates. A number found all the angles satisfying the tan ratio and all the angles satisfying the cos ratio and took the (only) common value. Others gave a number of answers for which they were penalised. In part(ii) the relationship between tan, sin and cos, a specific specification topic, was not well known. Squaring and using Pythagoras was of course an option but no one trying it this way got it right.
[(a) 216.9^0 (b) 116.6^0 and 296.6^0.]

Q6. (Binomial expansion)
A significant number of candidates did not expand using the binomial theorem, choosing instead to multiply out. This was of course acceptable, but long winded. Either way, a number were not able to use the fact that $\left(\sqrt{3}\right)^3 = 3\sqrt{3}$.

[$26 + 15\sqrt{3}$]

Q7. (Intersection of line and curve).
The usual errors were offered, most notably $y^2 = 12 - x^2 \Rightarrow y = \sqrt{12} - x$. Other errors included expanding $(3x + 1)^2 = 3x^2 + 1$.
It is worth noting here that the rubric requires answers to 3 significant figures unless otherwise stated. Candidates ought to be aware of this and be more careful in the way they give their answers. A number lost a mark here by not giving their answers to 2 decimal places as required.
[(-1.39, -3.17) and (0.79, 3.37)]

Q8. (Trigonometry - cosine and sine rules).
This was usually well done except for the calculation of the final bearing, for which a significant number of candidates lost a mark. Given the context of the question, 3 significant figures gives the nearest degree. We condoned one decimal place in this question but no more.
[308^0.]

Q9. (Binomial Probability)
The most significant error in this question was a failure to understand what “at least” means. It was expected that candidates would work 1 - P(0) - P(1). Some worked out the other 4 terms instead but a number gave P(2) as the answer.
[0.0815.]

Q10. (Factor Theorem)
Many found the connections between the parts difficult. Most candidates got part (i) correct but were then unable to use this in part (ii), preferring instead to start again with an attempt to solve a quadratic equation by the formula resulting in a negative discriminant and hence ho roots. This was perfectly acceptable but took rather longer than the award of one mark would warrant.
Likewise, part (iii) was done well but candidates were usually unable to use this in part (iv) to factorise and to find the quadratic that they had been dealing with in the first two parts.
[$(x + 3)^2 + 2$.]

Section B
Q11. (Linear programming)
There were many successful answers to this question, but also many who clearly had not covered this part of the syllabus. Once the inequalities had been derived and the graphs drawn the last part was straightforward, and it is quite possible that most candidates guessed the answer as, on this occasion, the objective function was not asked for.
((i) (9, 0)   (ii) (0, 12),   (iii)   (6, 6)]

Q12. (Coordinate geometry of the circle)
Candidates were not in general comfortable with this topic, and the derivation of the equation of the circle was not always successful. Candidates often also had difficulty finding the equation of the tangent. Many muddled the x and y axes, finding where this tangent cut the y axis.
[(a) $(x - 8)^2 + (y - 4)^2 = 25$ (b)(i) $3x + 4y = 65$, (b)(ii) 28.4 cm.]

Q13. (Algebra)
A significant number of candidates did not answer this question as they were unable to relate speed, distance and time. Many that might have done well then muddled the units, creating an equation involving 50 rather than $^5/_6$. However, many were able to reenter the question for the last part as the quadratic equation was given in the question, and this resulted in up to half marks.
[(i) $\frac{200}{v}$, $\frac{200}{v+20}$ $\frac{200}{v}$, $\frac{200}{v+20}$, (iii) 3 hrs 20 mins and 2 hrs 30 mins.]

Q14. (Calculus)
The question was generally well done except for two rather disappointing errors.
In part (b)(ii), candidates "lost" the denominator of 64. While this still gave $a = 0$ at $t = 8$ the acceleration function was not strictly correct.
In part (b)(iii) candidates "integrated" the denominator, giving the correct integrand divided by 64t.
[(a) (i) 2.5 ms^{-2}, (ii) 80 m, (b) (ii) 0, (iii) 80m.]

OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

ADDITIONAL MATHEMATICS 6993

Summer 2005

Monday **20 JUNE 2005** Morning 2 hours

Additional materials:
Answer booklet
Graph paper

TIME 2 hours

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Additional sheets of graph paper should be securely attached to your answer booklet.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given correct to three significant figures where appropriate.
- The total number of marks for this paper is 100.

This question paper consists of 5 printed pages and 3 blank pages.

Section A

1 Use calculus to show that there is a maximum point at $x = 3$ on the curve $y = 9x^2 - 2x^3$ and find the coordinates of this point. [5]

2 The function f(x) is defined by $f(x) = x^3 - 4x^2 + 5x - 2$.

(i) Find the remainder when f(x) is divided by $(x + 2)$. [2]

(ii) Show that $(x - 1)$ is a factor of f(x). [1]

(iii) Hence solve the equation $f(x) = 0$. [4]

3 A triangle has sides 8 cm, 7 cm and 12 cm. Calculate the largest angle of the triangle, correct to the nearest degree. [5]

4 Find the values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying the equation

$$4\sin\theta = 3\cos\theta.$$

Give your answers to the nearest 0.1 degree. [4]

5 In a large batch of glasses, 14% are defective. From this batch 8 glasses are selected at random. Calculate which is more likely:

(*A*) none of the glasses is defective,

(*B*) at least two of the glasses are defective. [7]

6 **(i)** Expand $\left(x - \frac{1}{x}\right)^4$ using the binomial expansion. Show all your working. [4]

(ii) Explain why the substitution $x = 1$ will help to justify your answer. [1]

7 The gradient function of a curve is given by $\frac{dy}{dx} = a + bx$. Find the values of a and b and the equation of the curve given that it passes through the points $(0, 2)$, $(1, 8)$ and $(-1, 2)$. [7]

8 A car moves in a straight line. Its velocity in metres per second, t seconds after passing a point A, is given by the equation

$$v = 27 - \frac{1}{8}t^3.$$

It comes to rest at a point B.

(i) Show that the car is at B when $t = 6$. [1]

(ii) Find the distance AB. [5]

9 **(i)** Using the identity $\cos^2\theta + \sin^2\theta = 1$, show that the equation

$$2\cos^2\theta + \sin\theta = 2$$

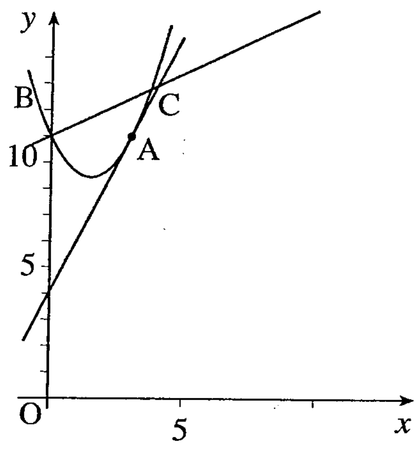
can be written as $2\sin^2\theta - \sin\theta = 0$. [2]

(ii) Hence find all values of θ in the range $0° \leqslant \theta \leqslant 180°$ satisfying the equation

$$2\cos^2\theta + \sin\theta = 2.$$ [4]

Section B

10



The curve shown has equation $y = \frac{2}{3}x^2 - 2x + 10$.

(i) Find the equation of the tangent to the curve at A (3, 10). [4]

(ii) Show that the equation of the normal to the curve at B (0, 10) is $2y - x = 20$. [3]

(iii) Find the coordinates of the point C where these two lines intersect. [3]

(iv) Calculate the length BC. [2]

11 A small factory makes two types of components, X and Y. Each component of type X requires materials costing £18 and each component of type Y requires materials costing £11. In each week materials worth £200 are available.

Each component of type X takes 7 man hours to finish and each component of type Y takes 6 man hours to finish. There are 84 man hours available each week.

Components cannot be left part-finished at the end of the week. In addition, in order to satisfy customer demands, at least 2 of each type are to be made each week.

(i) The factory produces x components of type X and y components of type Y each week. Write down four inequalities for x and y. [4]

(ii) On a graph draw suitable lines and shade the region that the inequalities do not allow. (Take 1 cm = 1 component on each axis.) [5]

(iii) If all components made are sold and the profit on each component of type X is £70 and on each component of type Y is £50, find from your graph the optimal number of each that should be made and the total profit per week. [3]

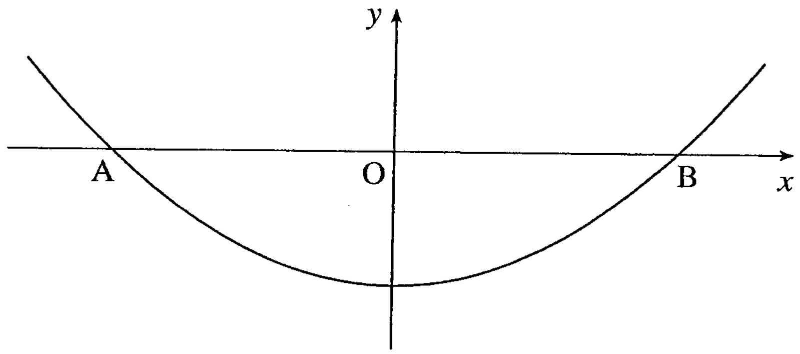
(Do not forget to hand in your graph paper with your answer booklet.)

12 **(i)** A circle has equation $x^2 + y^2 - 2x - 4y - 20 = 0$. Find the coordinates of its centre, C, and its radius. [3]

(ii) Find the coordinates of the points, A and B, where the line $y = x + 2$ cuts the circle. [5]

(iii) Find the angle ACB. [4]

13



The shape of the bed of a river is to be modelled mathematically. The diagram represents a cross-section of the river. A and B on the x-axis represent points on opposite banks of the river at water level. (Units are metres.)

The shape of the river bed between A and B is modelled by the equation

$$y = \frac{3}{16}(x^2 - 16).$$

(i) Find the coordinates of A and B and hence state the width of the river represented by the length AB. [2]

(ii) Find the depth of the river at its deepest point. [2]

(iii) Find the area of the cross-section of the river. [5]

(iv) The river flows at 20 metres a minute. You should assume that this rate applies to all points of this cross-section.

Find the volume of water that flows through this cross-section per minute. [1]

(v) Give two reasons why this model may not be a good model. [2]

OCR
RECOGNISING ACHIEVEMENT

June 2005

ADVANCED FSMQ

MARK SCHEME

Maximum mark: 100

Syllabus/component:

6993 Additional Mathematics

Paper Date: June 20, 2005

Mark Scheme

Section A

1		$\frac{dy}{dx} = 18x - 6x^2 = 0$ when $x^2 - 3x = 0$	M1	One term with decreasing power
		$\Rightarrow x(x-3) = 0 \Rightarrow x = 3, 0$	A1	Equates to 0 and gets 3 or subs 3 and gets 0
		$\frac{d^2y}{dx^2} = 18 - 12x$	DM1	Or sub $x = 3$ to give 0 Or "before and after" on gradient (or curve)
		When $x = 3$ $\frac{d^2y}{dx^2} < 0 \Rightarrow$ Maximum	A1	Getting it right and sub 3 and getting max.
		When $x = 3, y = 27$	B1	y coordinate
			5	

2	**(i)**	f(–2) = –8 – 16 – 10 – 2	M1	Or long division. We need to see $x^3 + 2x^2$ subtracted and x^2 on top.
		= –36	A1 **2**	
	(ii)	f(1) = 1–4 + 5 – 2 = 0	B1 **1**	Arithmetic must be seen
	(iii)	$\Rightarrow f(x) = (x-1)(x^2 - 3x + 2)$	M1	To find quadratic Or trial to find other roots
		$= (x-1)(x-1)(x-2)$	A1 A1	Each factor
		$\Rightarrow x = 1, 1, 2$	B1	Ans expressed properly
			4	

3		Largest angle is opposite longest side.	B1	Or for sum = 180
		$\cos\theta = \frac{8^2 + 7^2 - 12^2}{2 \times 8 \times 7}$	M1 F1	Give M1 F1 for cos rule regardless of which angle
		$= -0.2768 = \left(-\frac{31}{112}\right)$	A1	
		$\Rightarrow \theta = 106^\circ$	A1 **5**	
		Note 1: Other angles are 34.1 and 39.8 ***Note 2: Scale drawing could get the B1 but no more.***		See note above

4		$4\sin\theta - 3\cos\theta = 0 \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{3}{4} \Rightarrow \tan\theta = 0.75$	B1	For $\tan\theta = k$
		$\tan\theta = 0.75 \Rightarrow \theta = 36.9$, Also $\theta = 216.9$	M1 A1 F1	For 36.9 other value
		Alternatively: $16\sin^2\theta = 9\cos^2\theta \Rightarrow 16 - 16\cos^2\theta = 9\cos^2\theta$, $\Rightarrow \cos^2\theta = \frac{16}{25} \Rightarrow \cos\theta = 0.8$ $\Rightarrow \theta = 36.9,\ 216.9$	**4**	B1 M1 (use of pythag) A1 A1 (ignore extra values)

5		P(all sound) = $(0.86)^8 = 0.299$	B1	Allow 4 d.p.
		P(at least 2 defective) $= 1 - (0.86)^8 - 8(0.86)^7(0.14)$ $= 1 - \text{ans above} - 0.3896$ $= 0.311$	B1 M1 A1 M1 A1	powers coeff with term having product (0.3896 implies 8) for 1- 2 terms ans
		i.e. P(at least 2 defective) is greater.	F1 **7**	conclusion (based on their answers providing each is < 1

6	**(i)**	$\left(x - \frac{1}{x}\right)^4 = x^4 - 4x^3\frac{1}{x} + 6x^2\left(\frac{1}{x}\right)^2 - 4x\left(\frac{1}{x}\right)^3 + \left(\frac{1}{x}\right)^4$ $= x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4}$ *For complete expansion by multiplying, give B4, -1 for each mistake.*	B1 B1 B1 B1 **4**	powers correct coeffs correct signs outside brackets answer
	(ii)	Substituting will give the same value for both (= 0)	B1 **1**	

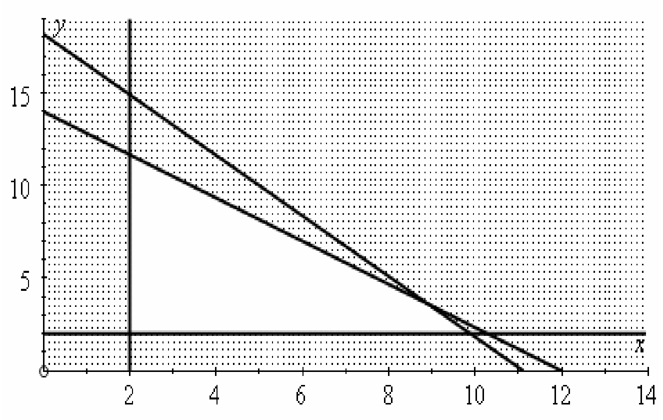
7		$\frac{dy}{dx} = a + bx \Rightarrow y = ax + \frac{bx^2}{2} + c$	M1 A1	Integrate (ignore *c*)
		$(0, 2) \Rightarrow c = 2$	M1 A1	Add and work out *c*.
		$(1, 8) \Rightarrow 8 = a + \frac{b}{2} + 2$	DM1	Attempt to substitute both
		$(-1, 2) \Rightarrow 2 = -a + \frac{b}{2} + 2$	M1	Solve simultaneously
		Solve: $b = 6, a = 3 \Rightarrow y = 3x + 3x^2 + 2$	A1 **7**	Equation must be given

8	**(i)**	$v=0$ when $\frac{1}{8}t^3=27 \Rightarrow t=2\times3=6$	B1 **1**	
	(ii)	$s=\int_0^6\left(27-\frac{1}{8}t^3\right)dt$ $=\left[27t-\frac{t^4}{32}\right]_0^6$ $=162-40.5=121.5$ Distance = 121.5 metres	M1 A1 M1 A1 B1 **5**	Integrate 2 correct terms Substitute Ans Units dependent on M marks

9	**(i)**	$\cos^2\theta=1-\sin^2\theta \Rightarrow 2-2\sin^2\theta+\sin\theta=2$ $\Rightarrow 2\sin^2\theta-\sin\theta=0$	M1 A1 **2**	N.B. Answer given
	(ii)	$2\sin^2\theta-\sin\theta=0 \Rightarrow \sin\theta(2\sin\theta-1)=0$ $\Rightarrow \sin\theta=0$ or $\sin\theta=\frac{1}{2}$ $\Rightarrow \theta=0,\ 30, 150, 180$	B1 B1 B1 B1 **4**	B1 for each angle all 4

Section B

10	**(i)**	$\frac{dy}{dx} = \frac{4x}{3} - 2$; When $x = 3$ $g = 2$ $\Rightarrow (y-10) = 2(x-3) \Rightarrow y = 2x+4$ (*Watch any use of (0,4) - no marks*)	M1 A1 DM1 A1 **4**	Diffn Correct line for their gradient
	(ii)	At (0,10) $g = -2$; Gradient of Normal = $\frac{1}{2}$ $\Rightarrow y = \frac{1}{2}x + 10 \Rightarrow 2y - x = 20$	M1 A1 B1 **3**	 g = –2 must be seen N.B. Answer given
	(iii)	Substitute: $2(2x+4) - x = 20 \Rightarrow 3x = 12$ $\Rightarrow x = 4,$ $y = 12$ (Allow 3 for (4, 12)	M1 A1 F1 **3**	 *y* value
	(iv)	Pythagoras for BC = $\sqrt{(12-10)^2 + (4)^2}$ $= \sqrt{20} = 2\sqrt{5} \approx 4.472$	M1 A1 **2**	

11	**(i)**	$18x + 11y \le 200$ $7x + 6y \le 84$ $x \ge 2$ $y \ge 2$	B1 B1 B1 B1 **4**	-1(once only) if = sign missing B0 for any line with inequality the wrong way round.
	(ii)	Graph plus shading 	B4 B1 **5**	B1 for each line correct shading, but only if all lines correct
	(iii)	Maximum profit is at the intersection of the lines: *P* = 70*x* + 50*y*. Nearest integer point to it is (9, 3) giving *P* = 780	B1 B1 B1 **3**	May be implied for any value (Intersection of lines is at (8.77, 3.76) giving 802.4)

12	**(i)**	$(x-1)^2+(y-2)^2=25$; Centre (1, 2), radius 5	M1 A1 F1 **3**	 For stating ans ft from equation
	(ii)	Substitute: $(x-1)^2+x^2=25$ $\Rightarrow 2x^2-2x-24=0$ $\Rightarrow x^2-x-12=0 \Rightarrow (x-4)(x+3)=0$ $\Rightarrow x=4,-3$ $\Rightarrow (4,6),\ (-3,-1)$	M1 F1 M1 A1 A1 **5**	ft from (i)
	(iii)	AC = BC = 5, AB = $7\sqrt{2}$ $\Rightarrow$ Angle = $2\times\sin^{-1}\left(\frac{\frac{7}{2}\sqrt{2}}{5}\right)=2\times\sin^{-1}(0.99)$ $=81.87 \Rightarrow \text{ACB}=163.7$	M1 A1 M1 A1 **4**	 for AB

13	**(i)**	A (-4, 0), B(4, 0) gives width = 8m	B1 B1 **2**	
	(ii)	x = 0, (0, -3) gives depth = 3m	B1 B1 **2**	ignore -ve sign
	(iii)	Cross section area = $\int_{-4}^{4}\frac{3}{16}\left(x^2-16\right)\mathrm{d}x$ $=\frac{3}{8}\int_{0}^{4}\left(x^2-16\right)\mathrm{d}x$ $=\frac{3}{8}\left[\frac{x^3}{3}-16x\right]_0^4=-\frac{3}{8}\left(\frac{128}{3}\right)$ $=16\,\mathrm{m}^2$	M1 M1 A1 A1 A1 **5**	Integration required. Correct expression (his limits from (i)) (ignore $^3/_{16}$) Integration Both terms Working it out - include $^3/_{16}$
	(iv)	$\Rightarrow$ Vol per minute = $16\times20=320\text{m}^3$	B1 **1**	20 × Ans from (iii)
	(v)	Water does not usually flow at a constant rate. The river bed will not be symmetric.	B1 B1 **2**	
		If units are omitted throughout, then subtract one mark. If not all parts have been done then deduct from first answer and bod the rest. ***If any of the answers have units then do not deduct the mark.***		

Examiner's Report

Free Standing Mathematics Qualification, Advanced Level
6993 Additional Mathematics

Summer, 2005
Chief Examiner's Report

The paper was a little easier this year than last, particularly in Section A. While this meant that a number of good candidates performed even better than the good candidates last year it is still true to say that there are a significant number of candidates who appear to have been entered for a qualification that is not suited to their abilities. For at least one centre no mark achieved reached double figures. The specification states that this is an Advanced FSMQ and that an appropriate starting point is a grade A*, A or B at GCSE with a thorough knowledge of the content of the Higher Tier. We believe that with this starting point a modest mark should be achievable with little or no extra learning. Consequently we believe that those achieving such low marks as single figures were not starting from this point and therefore this specification was not appropriate for them. This could not have been a positive experience for them and Centres might consider seeking advice as to what might be an appropriate course where they could demonstrate positive achievement.
The mean mark was 46.8.

Section A
Q1 (Calculus)
For most candidates this was a straightforward start to the paper, though many missed parts of the question, such as demonstrating that the turning point was a maximum or failing to find the y coordinate of the point.
[Maximum point at (3, 27)]

Q2 (Functions)
Most candidates knew the factor theorem and obtained the second part correctly, while many did not know the remainder theorem for part (i). Instead, long division was carried out, often successfully, but for many the inevitable algebraic errors crept in causing errors.
In part (iii) a few obtained the solution by trial and error, based on their knowledge of what possible roots there could be, given the constant number at the end. The significant majority used division to obtain a quadratic which they then solved correctly. Unfortunately a significant number of candidates failed to finish the question by giving the solution to the cubic equation, being content instead just to give the roots of the quadratic.
[(i) −36, (iii) $x = 1, 1, 2$]

Q3 (Cosine rule)
This question asked for the largest angle. Those who found it therefore gained full marks, but many of those who did not appreciate that the largest angle was opposite the longest side gave themselves extra work.
[106^0]

Q4 (Trigonometrical equation)
There were a few candidates who plotted the graphs $y = \sin x$ and $y = \cos x$ on their calculators and were able to zoom in on their intersections to find the roots to the required degree of accuracy; quite a number doing it this way, however, either failed to appreciate that there were two roots, or failed to find the values to the correct degree of accuracy, for which they were penalised.
The expected method was to obtain $\tan\theta = 0.75$. There were many errors here, including obtaining $\tan\theta = 1.33$.
The vast majority who failed to get anywhere with the question failed to demonstrate a knowledge of the relationship between the trigonometrical functions.

[36.9 and 216.9^0]

Q5 (Binomial distribution)
A few candidates did not read the question carefully. Some read the question as "exactly 2" rather than "at least 2". Many also failed to give a conclusion at the end.

Q6 (Binomial expansion)
This question was not always well done. The combination of the powers, the coefficients and the signs defeated all but the most able.
The last comment was often fudged and not at all clear, indicating that candidates did not appreciate that to make a substitution of a numerical value in the original and the final expansion should give the same value.
[$x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4}$]

Q7 (Integration)
This question was not done well and we were surprised by some of the answers. Even the most able candidates were integrating the function $\frac{dy}{dx} = a + bx$ to give $y = \frac{a^2}{2} + b\frac{x^2}{2}$. Even those who got it right then failed to appreciate that there was a non-zero constant of integration.
[$y = 3x + 3x^2 + 2$]

Q8 (Variable acceleration).
The first mark was easy to obtain for all candidates.
Part (ii) separated the candidates into those who knew that integration was required and those who assumed constant acceleration. A few candidates failed to give the answer properly, giving a numerical value for *s* only rather than stating the distance in metres.
[121.5 metres]

Q9 (Trigonometrical equation)
Only a few candidates failed to manipulate the equation using Pythagoras' theorem. However, the solution of the equivalent equation was not done nearly so well. The vast majority failed to appreciate that there are two roots from $\sin\theta = 0$ as well as two from $\sin\theta = 0.5$.
[0^0, 30^0, 150^0, 180^0]

Section B
Q10 (Coordinate geometry)
Part (i) was usually done well but a significant minority took a short cut by making unjustified assumptions from the diagram. In part(ii) there were more assumptions, this time occasionally justified (i.e. that the gradient of the tangent at (0, 10) is –2 given that the gradient at (3, 10) is 2). Candidates need to beware of making assumptions with no justification.
Parts (iii) and (iv) were usually well done and we saw occasionally candidates who were unable to complete parts (i) and (ii) reentering the question here.
The accuracy required for the answer to part (iv) was not given. Candidates should note the rubric of the paper and not simply write down every digit they see on their calculator. In these instances candidates should also be aware of the fact that an exact answer can be given and this was given full credit.
[(i) $y = 2x + 4$, (iii) (4, 12), (iv) $\sqrt{20} = 2\sqrt{5} = 4.472$]

Q11 (Linear programming)
This question was a source of a number of marks for weaker candidates. Practically every candidate realised that the “best” solution was not the intersection of the lines but a point nearest to it; most failed to investigate this with any logical order (by writing, for instance, a table), but many still got the right answer.
[(i) $18x + 11y \le 200$, $7x + 6y \le 84$, $x \ge 2$, $y \ge 2$, (iii) £780 at (9, 3)]

Q12 (Circle)
Those who knew something about the coordinate geometry of the circle got many marks in this question. Unfortunately there were many who were unable to manipulate the equation given into that required (by completing the square) to write down the radius and centre of the circle. Some were able to reenter the question in part (ii) and complete this part successfully. Those who did it most elegantly substituted into their rearranged equation. A few plotted the curve and the line and read off the points of intersections. We felt that this constituted an assumption again, and for full marks we required candidates to demonstrate that the integer points they picked out from their graph did satisfy both equations.
Part (iii) was testing for all candidates but a few were able to complete it successfully.
[(i) Centre (1, 2), radius 5, (ii) (4, 6) and (-3, -1) (iii) 163.7^0]

Q13 (Integration and modelling)
Parts (i) and (ii) were usually done quite well, though again, some good candidates lost marks by failing to answer the question properly. In part (i) they would leave it as A(–4, 0) and B(4, 0) without interpreting that the width was 8 metres. In part (ii) they would simply write $y = -3$ with no interpretation to give the greatest depth.
In part (iii) many got well into this integration but fell down over the manipulation of the fraction and brackets.
Part (iv) was well done even by those who got most of the first three parts wrong.
Many suggestions given in part (v) did not relate to the cross-section and some gave the same explanation in a different way.
Candidates who did not give correct units were penalised - in a modelling question the correct interpretation is required which is usually a little more than the numerical answer.
[(i) Width 8 m, (ii) Depth 3 m, (iii) 16 m^2, (iv) 320 m^3 (v) For e.g. the bed of a river is not usually smooth or symmetric, and the flow of water is not likely to be constant throughout the cross-section.]

OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

ADDITIONAL MATHEMATICS **6993**

Summer 2006

Thursday **15 JUNE 2006** Afternoon 2 hours

Additional materials:
16 page answer booklet
Graph paper

TIME 2 hours

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Additional sheets of graph paper should be securely attached to your answer booklet.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 100.

This question paper consists of 6 printed pages and 2 blank pages.

Section A

1 Find $\int_1^3 (x^2 + 3)\,dx$. [4]

2 Adam and Beth set out walking from a point P. After one hour Adam is 3.6 kilometres due north of P and Beth is 2.5 kilometres from P on a bearing of 035°.

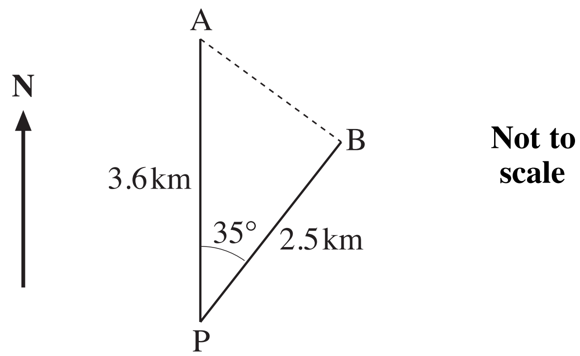


Fig. 2

Calculate how far they are apart at this time. Give your answer correct to 2 significant figures. [4]

3 Calculate the values of x in the range $0° < x < 360°$ for which $\sin x = 2\cos x$. [4]

4 **(i)** Find the distance between the points $(2, 3)$ and $(7, 9)$. [2]

(ii) Hence find the equation of the circle with centre $(2, 3)$ and passing through the point $(7, 9)$. [2]

5 Solve the inequality $x^2 + 4x > 5$. [5]

6 A curve has gradient given by $\frac{dy}{dx} = 2x + 2$. The curve passes through the point $(3, 0)$. Find the equation of the curve. [5]

7 **(i)** Show that the two lines whose equations are given below are parallel.

$$y = 4 - 2x$$
$$4x + 2y = 5$$
[2]

(ii) Find the equation of the line which is perpendicular to these two lines and which passes through the point $(1, 6)$. [3]

8 **(i)** By drawing suitable graphs on the same axes, indicate the region for which the following inequalities hold. You should shade the region which is **not** required.

$$3x + 2y \leqslant 18$$
$$y \leqslant 3x$$
$$y \geqslant 0$$
[5]

(ii) Find the maximum value of $x + 2y$ subject to these conditions. [2]

9 You are given that $\mathrm{f}(x) = x^3 - 4x^2 + x + 6$.

(i) Find the remainder when $\mathrm{f}(x)$ is divided by $(x - 1)$. [1]

(ii) Show that $(x - 3)$ is a factor of $\mathrm{f}(x)$. [2]

(iii) Hence solve the equation $\mathrm{f}(x) = 0$. [4]

10 Find the coordinates of the points of intersection of the line $y = 5 - 2x$ with the curve $y = x^2 - 4x - 11$, giving your answers correct to 2 decimal places. [7]

Section B

11 It is known that 65% of all people living in the UK went abroad for a holiday last year.

A random sample of 5 people living in the UK was chosen.

Find the probability that

(i) all 5 went abroad for a holiday last year, [1]

(ii) exactly 4 went abroad for a holiday last year, [3]

(iii) at least 2 went abroad for a holiday last year. [4]

An additional random sample of 5 people living in the UK was chosen.

(iv) Find the probability that in the 10 people chosen altogether, exactly 8 went abroad for a holiday last year. [4]

12 A train normally travels between two points A and D at a constant speed of 30 metres per second. The distance AD is 12 kilometres.

(i) Find the time taken for the train to travel between A and D at $30\,\mathrm{m\,s^{-1}}$. [1]

Between A and D there are two other points, B and C, which are placed such that AB = 2 km, BC = 6 km and CD = 4 km. On one day there is a speed restriction of $10\,\mathrm{m\,s^{-1}}$ between B and C.

The train decelerates uniformly from $30\,\mathrm{m\,s^{-1}}$ at A to $10\,\mathrm{m\,s^{-1}}$ at B. It travels the distance BC at $10\,\mathrm{m\,s^{-1}}$. The train then accelerates uniformly from $10\,\mathrm{m\,s^{-1}}$ at C to $30\,\mathrm{m\,s^{-1}}$ at D.

Find

(ii) the time taken to travel from A to B, [2]

(iii) the acceleration over the distance CD, [3]

(iv) the extra time taken in travelling from A to D as a result of the speed restriction. [6]

13 Fig. 13.1 shows a solid block which is in the shape of a pyramid. The horizontal base, ABCD, is a square with side 20 cm and the vertex, V, is 15 cm vertically above the centre, O, of the square base. N is the midpoint of AB.

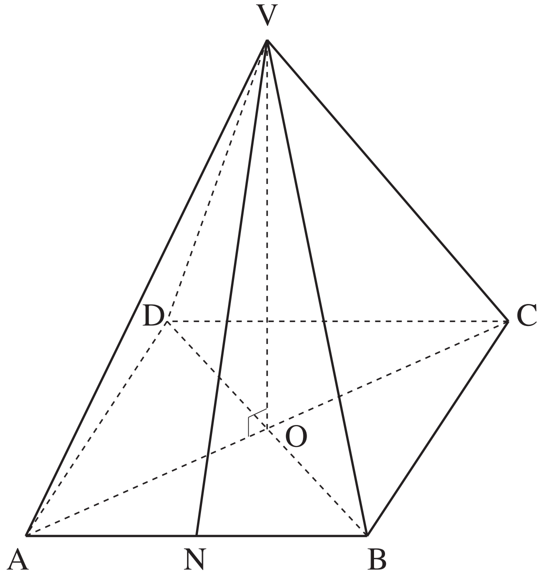


Fig. 13.1

(i) Calculate the length of the diagonal AC. [2]

(ii) Show that the length of the edge AV is $\sqrt{425}$ cm. [2]

(iii) Calculate the angle that the edge AV makes with the base. [2]

(iv) Calculate the length VN. [2]

M is the point on VB such that AM is perpendicular to VB as shown in Fig. 13.2.

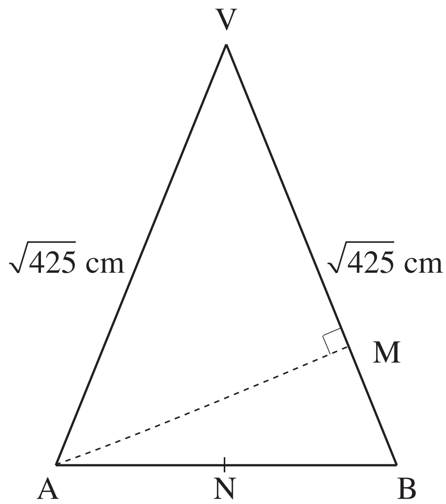


Fig 13.2

(v) Calculate the area of triangle VAB. Hence or otherwise calculate the distance AM. [4]

[Turn over

14

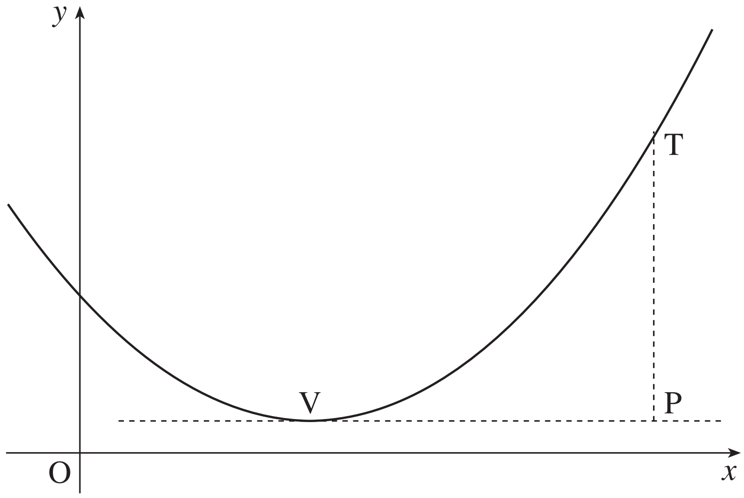


Fig. 14

Fig. 14 shows the quadratic curve $y = x^2 - 4x + 5$.

V(2, 1) is the minimum point of the curve.

T(5,10) is a point on the curve.

The line VP is the tangent to the curve at V and TP is perpendicular to this line.

(i) Write down the coordinates of P. [1]

(ii) Find the coordinates of M, the midpoint of VP. [2]

(iii) Find the equation of the tangent to the curve at T. [4]

(iv) Show that the tangent to the curve at T passes through the point M. [2]

(v) Use the result in part **(iv)** to suggest a way of drawing a tangent to a point on a quadratic curve without involving calculus. [3]

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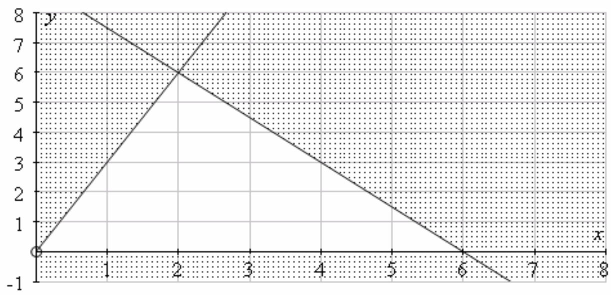
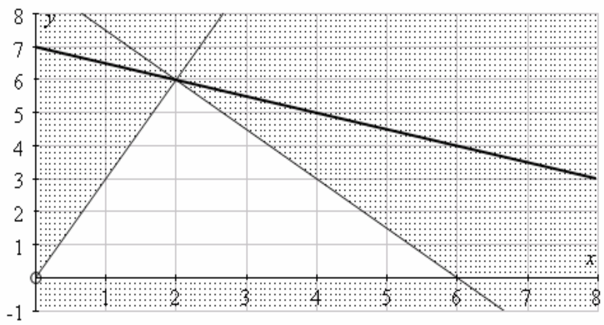
6993 Additional Mathematics. June 2006 Final Mark scheme

Q.		Answer	Mark	Notes
Section A				
1		$\int_1^3 (x^2+3)\,dx = \left[\frac{x^3}{3}+3x\right]_1^3$	M1 A1	Attempt to integrate Both terms correct
		$=\left(\frac{27}{3}+9\right)-\left(\frac{1}{3}+3\right)$	DM1	Substitute 3 and 1 and subtract
		$=18-3\frac{1}{3}=14\frac{2}{3}$ Accept 14.7 but not 14.6	A1 **4**	Sp. Case: Allow M1 even if one of the values is not as per question.
2		Cosine rule: $AB^2 = 2.5^2+3.6^2-2\times2.5\times3.6\times\cos 35$	M1 A1	Attempt at cosine rule plus sub I their formula (May have + and/or no 2) May be implied
		$=4.465.....$ $\Rightarrow AB = 2.1$ (km)	A1 A1 **4**	A0 if correct ans not given to 2.s.f. (This counts as the tfsf penalty for the paper.)
		Alternatively: Find sides of rt angled triangle putting East line across triangle, then use trig and Pythagoras is OK		
3		$\sin x = 2\cos x \Rightarrow \tan x = 2$ $\Rightarrow x = 63.4,$ Add 180 243 (allow 243.4)	B1 B1 M1 F1	for 63.4 For adding 180 (and nothing else Not if extras are in range.
		Alternative: $\sin x = 2\cos x \Rightarrow \sin^2 x = 4\cos^2 x \Rightarrow 1-\cos^2 x = 4\cos^2 x$		B1
		$\Rightarrow \cos^2 x = \frac{1}{5} \Rightarrow \cos x = \pm\frac{1}{\sqrt{5}} \Rightarrow x = 63.4$		B1
		Sorting which quadrant for other root $\Rightarrow$ Add 180 243 (allow 243.4)	**4**	M1 F1
4	(i)	$d=\sqrt{(7-2)^2+(9-3)^2}$ $=\sqrt{25+36}=\sqrt{61}$ $(=7.81)$	M1 A1 **2**	
	(ii)	$(x-2)^2+(y-3)^2=61$	M1 F1 **2**	Correct LHS = something

5		$x^2+4x>5 \Rightarrow x^2+4x-5>0$ $\Rightarrow (x+5)(x-1)>0$ Both positive or both negative $\Rightarrow x>1$ or $x<-5$ Sp. Case Sub $x = 1$ gives f(x) = 0 gives $x > 1$ B1 Sub $x = -5$ gives f(x) = 0 gives $x < -5$ B1	M1 M1 A1 A1 A1 **5**	Get Quad Factorise or sketch LHS: Allow $(x + 2)^2 > 9$ Can be obtained from sketch or drawn on a number line on sketch.
6		$\frac{\mathrm{d}y}{\mathrm{d}x}=2x+2 \Rightarrow y=x^2+2x+c$ Sub: $0=9+6+c \Rightarrow c=-15$ $\Rightarrow y=x^2+2x-15$	M1 A1 A1 DM1 A1 **5**	Integrate For $x^2 + 2x$ Includes c Must be $y = \ldots\ldots$
7	(i)	First line $y = -2x + 4$ 2nd line: $y = -2x + {}^5/_2$ Therefore same gradients (Alt. Try to solve and get impossibility such as 8 = 5)	B1 B1 **2**	Both values seen clearly to be -2 Comment
	(ii)	Perpendicular line has gradient $\frac{1}{2}$ $\Rightarrow y-6=\frac{1}{2}(x-1) \Rightarrow 2y-x=11$	M1 DM1 A1 **3**	For negative reciprocal (must be numeric) in any equivalent form
8	(i)		B1 B1 B1 B1 B1 **5**	$3x + 2y \le 18$ Shading $y = 3x$ Shading $y \ge 0$ shading -1 if triangle shaded
	(ii)	 Point required is intersection this is (2, 6) giving 14	M1 A1 **2**	Includes attempt to work out $x + 2y$

9	**(i)**	f(1) = 1 – 4 + 1 + 6 = 4	B1 **1**	
	(ii)	f(3) = 27 – 36 + 3 + 6 = 0 i.e. $(x – 3)$ is a factor	M1 A1 **2**	Substitute or long division. If latter then we must see $x^3 – 3x^2$
	(iii)	$x^3 - 4x^2 + x + 6 = 0$ $\Rightarrow (x-3)(x^2 - x - 2) = 0$ $\Rightarrow (x-3)(x-2)(x+1) = 0$ $\Rightarrow x = -1, 2, 3$ Attempt to get quadratic can be by "trial" or long division Alt: test for a root consistent with 6 M1 Get one root A1 Get the other root A1 Give answer F1	M1 A1 A1 F1 **4**	Attempt to factorise Quadratic Factors But only if integers i.e. x = -1, ± 2
10		Substitute: $y = 5 - 2x \Rightarrow 5 - 2x = x^2 - 4x - 11$ $\Rightarrow x^2 - 2x - 16 = 0$ $\Rightarrow x = 5.12, -3.12$ $\Rightarrow (5.12, -5.25), (-3.12, 11.25)$ Special cases: Graphs B1 B1 M1(acknowledging that the answer is where they meet) Max: 3 If no graph but points are given then B1, B1 for each pair. N.B. It is possible to eliminate x to give a quadratic is y. This is $y^2 – 16y – 109 = 0$	M1 A1 M1 A1 F1 M1 A1 **7**	Correct sub For quadratic eqn Solve Each x Correct pairing

Q.		Answer	Mark	Notes
Section B				
11	**(i)**	$(0.65)^5 = 0.1160$	B1 **1**	
	(ii)	$5(0.65)^4(0.35) = 0.3124$	B1 B1 B1 **3**	$(0.65)^4(0.35)$ $5\times$ Ans
	(iii)	$1-(0.35)^5-5(0.35)^4(0.65)$ $=1-0.00525-0.0488$ $=0.9460$ Misread Sp. Case: Adding 0, 1 and 2 M1 A1 A1 A0 but – 1 misread. Alt: Add terms = P(2) + P(3) + P(4) + P(5) Add 4 binomial = 0.1812 + 0.3364 terms M1 + 0.31324 + 0.1160 Powers A1 Coeffs A1 =0.946 Ans A1	M1 A1 A1 A1 **4**	1 – 2 or 3 binomial terms Powers (all correct) Coeff ans
	(iv)	$\begin{pmatrix}10\\8\end{pmatrix}(0.65)^8(0.35)^2 = 0.1757$	M1 A1 A1 A1 **4**	Binomial term with at least sum of powers = 10 Powers Coeff Ans

Q.		Answer	Mark	Notes
12	(i)	$t=\frac{12000}{30}=400$ sec	B1 **1**	
	(ii)	$s=\frac{(u+v)}{2}t \Rightarrow t=\frac{2\times 2000}{10+30}=100\text{sec}$	M1 A1 **2**	
	(iii)	$v^2=u^2+2as \Rightarrow 30^2=10^2+2a.4000$ $\Rightarrow a=\frac{800}{8000}=\frac{1}{10}=0.1\text{ms}^{-2}$	M1 A1 A1 **3**	For any valid const accel formula Credit 2 for t_3 if seen here.
	(iv)	$s=\frac{(u+v)}{2}t \Rightarrow t=\frac{2\times 4000}{10+30}=200\text{sec}$ For 2nd part: $s=vt \Rightarrow t=\frac{6000}{10}=600\text{sec}$ $\Rightarrow$ total time = 900 sec. Original time = 400 sec so loss is 500 secs	M1 A1 M1 A1 M1 F1 **6**	Can be given in (ii) if seen. For $t_1+t_2+t_3$ – their (i)

N.B. Using km throughout counts as misread.
Using 100 m = 1 km is also a misread.
Make sure they are consistently wrong throughout the paper. If not, then deduct the appropriate marks

13	**(i)**	$AC=\sqrt{20^2+20^2}=20\sqrt{2}\approx 28.3$	M1 A1 **2**	
	(ii)	$AV=\sqrt{15^2+200}=\sqrt{425}$	M1 A1 **2**	Must be clear (N.B. Ans given)
	(iii)	$\text{Angle VAO}=\tan^{-1}\dfrac{15}{14.14}\approx 46.7^0$	M1 A1 **2**	Using half (i) and tan
	(iv)	$VN=\sqrt{425-100}=\sqrt{325}\approx 18.0$ $OR=\sqrt{15^2+10^2}=\sqrt{325}\approx 18.0$	M1 A1 **2**	
	(v)	$\text{Area}=\frac{1}{2}AB.VN=\frac{1}{2}20\sqrt{325}\approx 180.2....$ $\text{Area}=\frac{1}{2}AM.VB=\frac{1}{2}AM\sqrt{425}$ $\Rightarrow \frac{1}{2}20\sqrt{325}=\frac{1}{2}AM\sqrt{425}$ $\Rightarrow AM=\dfrac{20\sqrt{325}}{\sqrt{425}}\approx 17.5$ N.B. Candidates might find AM by other means and then find the area of the triangle using AM. This is acceptable	M1 A1 M1 A1 **4**	Either form Finding angle B and then AM from triangle AMB M1A1 And then area can be found M1 A1

14	**(i)**	P(5, 1)	B1 **1**	
	(ii)	M is $\left(\dfrac{2+5}{2},1\right)=\left(3\frac{1}{2},1\right)$	M1 F1 **2**	2 + their (i), divided by 2
	(iii)	$y=x^2-4x+5\Rightarrow\dfrac{dy}{dx}=2x-4$ When $x=5,\ g=6\Rightarrow y-10=6(x-5)\Rightarrow y=6x-20$ $\Rightarrow y=6x-20$	M1 A1 DM1 A1 **4**	Differentiate finding g and using eqn of line
	(iv)	Substitute the coordinates of M into line When $x=3.5, y=6\times3.5-20=21-20=1$	DM1 A1 **2**	Only if line and point are correct!
	(v)	Find the minimum point and draw a line parallel to the x-axis Drop a perpendicular from T to this line at P. Find the midpoint of VP, M The tangent goes through T and M.	B1 B1 B1 **3**	

Free Standing Mathematics Qualification, Advanced Level.
6993 Additional Mathematics

Summer 2006
Chief Examiner's Report

The number of candidates for this specification continues to rise, with an entry nearly 15% up from last year and almost double the entry for the first examination in 2003.
We were pleased to see a large number of very good scripts - in more than one centre the total candidature recorded a mark of over 80%. However, it is still disappointing to find a number of centres for which this specification is clearly not appropriate. The specification clearly states that the specification is suitable for those gaining a good grade at GCSE - typically A*, A or B. The specification is designed to be an enrichment programme for Higher Tier students and it is therefore inappropriate for an entry for students at any other level.

The rubric states that answers should be given to 3 significant figures where appropriate. In past years this has resulted in marks being deducted for the following reasons
Answers being approximated to less than 3 significant figures, particularly the answers in the binomial probability question
Angles being given to 2 or more decimal places
Lengths being given to a large number of significant figures, usually resulting from candidates writing down the total display on their calculator.

The "appropriateness" of this procedure should be evident in questions 2 (where 2 significant figures was demanded) 4, 10, 11 and 13. In general, we adopted a policy of deducting a mark for this where it was first seen and only once throughout the paper.

Section A

Q1 (Calculus)

Better candidates had few problems, though the "integration" of the second term to give $\frac{3^2}{2}$ was often seen. Even those who got the integration correct failed to complete the arithmetic correctly; typically we saw $\frac{3^3}{3} = 3$.

Q2 (Cosine rule)
There was an alternative method of course, which was to draw a line East -West from B, calculating the sides of the two resulting right-angled triangles. This was a typical situation where candidates lost time due to working through a process that was rather longer than the expected method.
Of those who used the cosine rule, some failed to remember the formula properly and many failed to give the answer to 2 significant figures as required.
A large number of candidates also left their answer as 4.465… which is a^2, in spite of writing the formula correctly, and so lost the last accuracy mark for failing to take the square root.

Q3 (Trigonometry)
This was attempted by a variety of methods, most leading to inaccurate values. Trial and improvement should be discouraged with this work as it is both time consuming and unnecessary. Most who obtained the first value were also able to give the second and only a very few found values in other quadrants.

Q4 (Coordinate geometry of the circle)
While the vast majority of candidates were able to evaluate the distance between two points, dealing with the equation of a circle which did not have its centre at the origin was not at all well known.

Q5 (Inequalities)
About a third of candidates did not understand that they had to factorise a quadratic function to proceed with the question. Most of the remainder were able to deal with the correct factorisation, but unable to complete the inequality. A common answer was $x > 1$ and $x > -5$.

Q6 (Calculus)
Some omitted the constant of integration then spuriously tried to compensate thereafter. Only a few replaced the m in the general equation of the line by the function of x given as the gradient function.

Q7 (Coordinate geometry)
There were two acceptable methods. The first was to write both equations in the form $y = mx + c$ and to comment that the coefficient of x, which is the gradient, is the same for both lines. Of those who did this a large number said that the gradient was $-2x$. The other method was to claim that two lines are parallel if they do not intersect and attempts to find the point of intersection by solving simultaneously would, for two parallel lines, produce an impossibility (typically $8 = 5$). This is quite subtle and unfortunately we were not convinced in most cases that candidates knew this and were trying to develop this argument. They solved simultaneously (perhaps because they did not know what else to do) and then could not cope with the apparent mess into which they were getting. The gradient of the perpendicular line seemed to be well known and those who found -2 as the common gradient used $\frac{1}{2}$ as the gradient of a perpendicular line successfully to complete the question. Of those who wrote the gradient of the given lines as $-2x$ some then wrote the gradient of a perpendicular line as $\frac{1}{2x}$. Some successfully completed the question, and so we put this down to sloppy notation but others got themselves confused.

Q8 (Linear Programming)
In general this question was well done. Common errors that led to the loss of one or more marks were:

- The incorrect shading for the inequality $y \leq 3x$ which not only led to the incorrect answer but encouraged candidates to shade incorrectly also the domain $y \geq 0$, shading instead the region for which $x \geq 0$.
- The drawing of the line $y = \frac{1}{3}x$
- The final answer left as (2, 6).

Q9 (Polynomials)
It was clear that answers to this question were more than usually centre-dependent, in that in some centres hardly any candidate got it right and in many centres practically every candidate obtained full marks.
Most candidates were able to justify that $(x - 3)$ was a factor by using the factor theorem (though many did not say so, simply showing that $f(3) = 0$ with no comment) but a significant number of these did not seem to know the remainder theorem and obtained the answer to (i) by long division. Rather more candidates than last year gave the full solution to the equation, though some did still give $x = -1, 2$ as the answer.

Q10 (Intersection of line and curve)
A few candidates failed to substitute properly and their algebraic manipulation let them down. Most were able to solve their quadratic equation, however. Once again, marks were lost, often by very good candidates, by failing to read the question. In this question the y values were required as well.

Section B

Q11 (Binomial distribution)
Most candidates knew what to do but there were the expected few who failed to write terms which had consistent powers or coefficients. A surprising number worked with the probabilities 0.65 and 0.45 or even 0.25.

Q12 (Constant acceleration)
There were very few candidates who were unable to make any headway with this question. However, the constant acceleration formulae were not well known; many used $u = 0$ throughout and also many failed to use average speed during the sections of deceleration and acceleration.
For those who used a formula requiring a time in (iii) the two marks allocated to this in the mark scheme (and on the paper) in (iv) were awarded when seen in (iii). For these candidates the allocation of marks to the sections was 1, 2, 5, 4.
A number of candidates used kilometres and some also took 100 m = 1 km. If one of these errors had been consistent throughout the question it would have been possible to treat it as a misread, but unfortunately many of these candidates used incorrect units or conversion inconsistently, dealing with it correctly in some parts but not in others.

Q13 (3-D trigonometry)
This question was possibly the best of the section B questions, perhaps because it was nearest to being part of the GCSE syllabus. In (ii) many answers were unconvincing. Candidates should be

clear that when a question says "show" then no fudging or omission of working is acceptable. In this case also it was not acceptable to take an approximate value to be rounded to the given value. A handful found the wrong angle in (iii). Others used their angle in (iii) in part (v). Generally though, apart from (v), this was popular and an easy source of marks for most of the candidates. In some cases this was the only significant source of marks.
The straightforward method of answering (v) was not adopted by most candidates who chose a rather more complicated route to get to the answers. Finding AM in order to evaluate the area was accepted.

Q14 (Calculus of curves)
Better candidates had few problems and seemed to do the whole problem in a few lines.
The majority were able to score full marks in (i) and (ii). Some differentiated in (iii) then stopped, others read ahead and worked out the equation of the line TM rather than the tangent. Some of the descriptions in (v) were vague, but attempts to describe what had been done in this specific case as a general process were credited.

FSMQ Advanced Additional Mathematics 6993
June 2006 Assessment Session

Unit Threshold Marks

Unit	Maximum Mark	A	B	C	D	E	U
6993	100	79	67	56	45	34	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
6993	35.2	48.1	57.3	65.7	75.3	100	4381

RECOGNISING ACHIEVEMENT

FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

ADDITIONAL MATHEMATICS **6993/01**

THURSDAY 14 JUNE 2007

Afternoon
Time: 2 hours

Additional materials:
Answer booklet (16 pages)
Graph paper

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 100.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

Section A

1 Solve the inequality $3(x+2) > 2 - x$. [3]

2 A particle moves in a straight line. Its velocity, $v\,\text{m s}^{-1}$, t seconds after passing a point O is given by the equation

$$v = 6 + 3t^2.$$

Find the distance travelled between the times $t = 1$ and $t = 3$. [4]

3 A circle has equation $x^2 + y^2 - 4x - 6y + 3 = 0$.

Find the coordinates of the centre and the radius of the circle. [3]

4 Find all the values of x in the range $0° < x < 360°$ that satisfy $\sin x = -4\cos x$. [5]

5 A car is travelling along a motorway at $30\,\text{m s}^{-1}$. At the moment that it passes a point A the brakes are applied so that the car decelerates with constant deceleration. When it reaches a point B, where AB = 300 m, the speed of the car is $10\,\text{m s}^{-1}$.

Calculate

(i) the constant deceleration, [3]

(ii) the time taken to travel from A to B. [2]

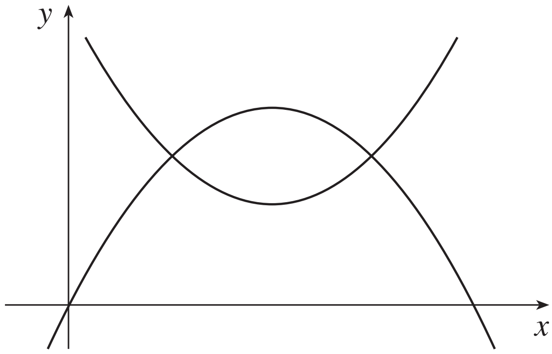
6 Find the equation of the tangent to the curve $y = x^3 - 3x + 4$ at the point $(2, 6)$. [4]

7 Use calculus to find the x-coordinate of the minimum point on the curve

$$y = x^3 - 2x^2 - 15x + 30.$$

Show your working clearly, giving the reasons for your answer. [7]

8 The figure shows the graphs of $y = 4x - x^2$ and $y = x^2 - 4x + 6$.



(i) Use an algebraic method to find the x-coordinates of the points where the curves intersect. [3]

(ii) Calculate the area enclosed by the two curves. [4]

9 The points A, B and C have coordinates $(-1, 1)$, $(5, 8)$ and $(8, 3)$ respectively.

(i) Show that AC = AB. [2]

(ii) Write down the coordinates of M, the midpoint of BC. [1]

(iii) Show that the lines BC and AM are perpendicular. [2]

(iv) Find the equation of the line AM. [2]

10 **(i)** By drawing suitable graphs on the same axes, indicate the region for which the following inequalities hold. You should shade the region which is **not** required.

$$\begin{aligned} 2x + 3y &\leqslant 12 \\ 2x + y &\leqslant 8 \\ y &\geqslant 0 \\ x &\geqslant 0 \end{aligned}$$

[5]

(ii) Find the maximum value of $x + 3y$ subject to these conditions. [2]

Section B

11 **(a)** You are given that $f(x) = x^3 - 3x^2 - 4x$.

(i) Find the three points where the curve $y = f(x)$ cuts the x-axis. [4]

(ii) Sketch the graph of $y = f(x)$. [1]

(b) You are given that $g(x) = x^3 - 3x^2 - 4x + 12$.

(i) Find the remainder when $g(x)$ is divided by $(x + 1)$. [2]

(ii) Show that $(x - 2)$ is a factor of $g(x)$. [1]

(iii) Hence solve the equation $g(x) = 0$. [4]

12 The work-force of a large company is made up of males and females in the ratio 9 : 11. One third of the male employees work part-time and one half of the female employees work part-time.

8 employees are chosen at random.

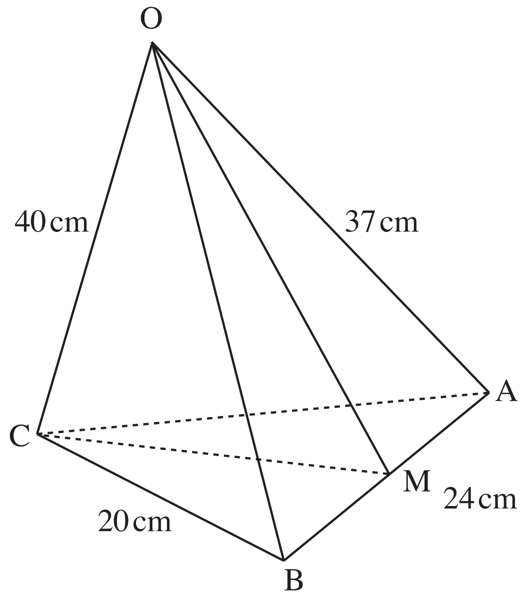
Find the probability that

(i) all are males, [2]

(ii) exactly 5 are females, [4]

(iii) at least 2 work part-time. [6]

13 In the pyramid OABC, OA = OB = 37 cm, OC = 40 cm, CA = CB = 20 cm and AB = 24 cm. M is the midpoint of AB.



Calculate

(i) the lengths OM and CM, [3]

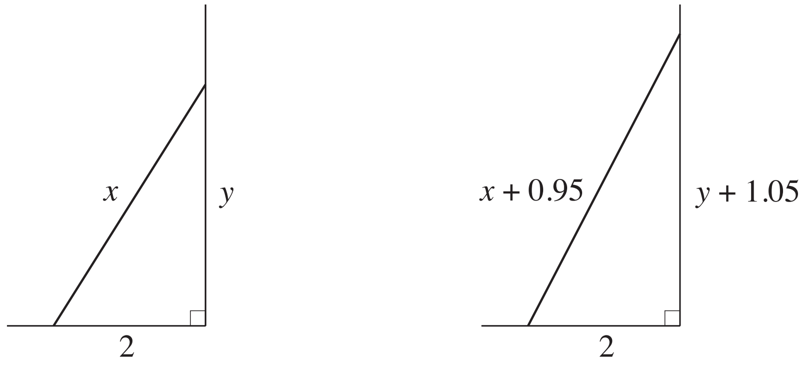
(ii) the angle between the line OC and the plane ABC, [4]

(iii) the volume of the pyramid. [5]

[The volume of a pyramid $= \frac{1}{3} \times$ base area $\times$ height.]

[Question 14 is printed overleaf.]

14 An extending ladder has two positions. In position **A** the length of the ladder is x metres and, when the foot of the ladder is placed 2 metres from the base of a vertical wall, the ladder reaches y metres up the wall.



Position A **Position B**

In position B the ladder is extended by 0.95 metres and it reaches an extra 1.05 metres up the wall.

The foot of the ladder remains 2 m from the base of the wall.

(i) Use Pythagoras' theorem for position **A** and position **B** to write down two equations in x and y. [2]

(ii) Hence show that $2.1y = 1.9x - 0.2$. [3]

(iii) Using these equations, form a quadratic equation in x.

Hence find the values of x and y. [7]

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RECOGNISING ACHIEVEMENT

FSMQ

Additional Mathematics

ADVANCED FSMQ **6993**

Mark Scheme for the Unit

June 2007

6993/MS/R/07

Oxford Cambridge and RSA Examinations

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CONTENTS

Additional Mathematics FSMQ (6993)

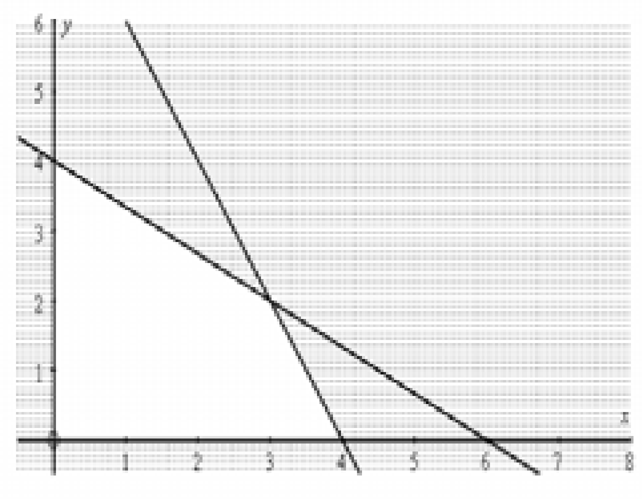
MARK SCHEME FOR THE UNIT

Mark Scheme 6993
June 2007

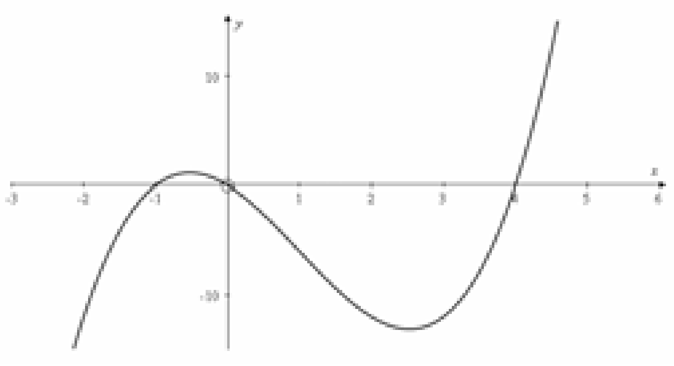
Q.		Answer	Mark	Notes
Section A				
1		$3(x+2) > 2 - x$ $\Rightarrow 3x + 6 > 2 - x$ $\Rightarrow 4x > -4$ $\Rightarrow x > -1$	M1 A1 A1 **3**	Expand and collect Only 2 terms
2		$v = 6 + 3t^2 \Rightarrow s = 6t + t^3 + c$ Take $s = 0$ when $t = 1 \Rightarrow c = -7$ When $t = 3,\ s = 18 + 27 - 7 = 38$ Alternatively: $s = \int_1^3 (6 + 3t^2)\,dt = \left[6t + t^3\right]_1^3 = (18+27) - (6+1) = 38$	M1 A1 DM1 A1 **4**	Ignore *c* Either sub to find *c* or sub and subtract from definite integral M1 int A1 DM1 sub and sub A1
3		$x^2 + y^2 - 4x - 6y + 3 = 0$ $\Rightarrow x^2 - 4x + y^2 - 6y = -3$ $\Rightarrow x^2 - 4x + 4 + y^2 - 6y + 9 = 4 + 9 - 3$ $\Rightarrow (x-2)^2 + (y-3)^2 = 10$ $\Rightarrow$ Centre (2, 3), radius $\sqrt{10}$ $(\approx 3.162...)$ SC: Penultimate line M1 A1 S.C. Centre B1 Find a point on the circle and then use Pythagoras M1 to find radius A1	M1 B1 A1 **3**	Complete the square Centre Radius *Accept correct answers with no working*
4		$\sin x = -4\cos x \Rightarrow \tan x = -4$ $\Rightarrow x = \pm 75.96^0$ $\Rightarrow x = 180 - 75.96 = 104^0$ and $x = 360 - 75.96 = 284^0$ Alternatively Use of $s^2 + c^2 = 1$ M1 $\Rightarrow \cos^2 x = \frac{1}{17}$ $\Rightarrow x = \pm 75.96^0$ A1 $\Rightarrow x = 180 - 75.96 = 104^0$ M1 A1 and $x = 360 - 75.96 = 284^0$ A1 S.C. Graphical method $\pm 2^0$ tolerance B1 B1 S.C. Answers with no working B1 for both.	B1 B1 M1 A1 A1 **5**	For either value from calculator For method to find a correct answer from that given on calculator -1 extra values Ignore values outside 360^0

5	(i)	Using $v^2 = u^2 + 2as$ $\Rightarrow 10^2 = 30^2 + 2a.300$ $\Rightarrow 600a = -800$ $\Rightarrow a = -\frac{4}{3}$	M1 A1 A1 **3**	Got to be used! *Ignore –ve sign.*
	(ii)	Using $v = u + at$ $\Rightarrow 10 = 30 - \frac{4}{3}t$ $\Rightarrow t = 20 \times \frac{3}{4} = 15$ Or: $s = \frac{u+v}{2}t$ $\Rightarrow 300 = \frac{30+10}{2}t$ $\Rightarrow t = \frac{600}{40} = 15$	M1 F1 **2**	From their *a* This could be used in (i) to find *t* then *a*
6		$\frac{dy}{dx} = 3x^2 - 3$ At (2, 6) $\frac{dy}{dx} = 9 \Rightarrow y - 6 = 9(x-2)$ $\Rightarrow y = 9x - 12$	B1 M1 DM1 A1 **4**	Diff correctly Substitute in their gradient function Set up equation with their gradient
7		$\frac{dy}{dx} = 3x^2 - 4x - 15$ $= 0$ when $3x^2 - 4x - 15 = 0$ $\Rightarrow (3x+5)(x-3) = 0$ $\Rightarrow x = 3, -\frac{5}{3}$ $\frac{d^2y}{dx^2} = 6x - 4$: When $x = 3$, $\frac{d^2y}{dx^2} > 0$ $\Rightarrow$ minimum $\Rightarrow x = 3$ N.B. Any valid method is acceptable, but not that *x* = 3 is the right hand value or that the *y* value is lower then for the other value of *x*.	M1 A1 M1 A1 M1 F1 A1 **7**	=0 and attempt to solve Differentiate again and substitute Providing all other marks earned

8	(i)	$4x-x^2=x^2-4x+6$ $\Rightarrow 2x^2-8x+6=0$ $\Rightarrow x^2-4x+3=0$ $\Rightarrow (x-3)(x-1)=0$ $\Rightarrow x=3,1$	M1 M1 A1 **3**	Equate and attempt to collect terms Solve a quadratic Ans only seen - B1
	(ii)	Area = $\int_1^3 (4x-x^2)\,dx - \int_1^3 (x^2-4x+6)\,dx$ $=\left[2x^2-\frac{x^3}{3}\right]_1^3-\left[\frac{x^3}{3}-2x^2+6x\right]_1^3$ $=(18-9)-\left(2-\frac{1}{3}\right)-(9-18+18)+\left(\frac{1}{3}-2+6\right)$ $=9-1\frac{2}{3}-9+4\frac{1}{3}=2\frac{2}{3}$ Alternatively: Area = $\int_1^3 (8x-2x^2-6)\,dx$ $=\left[4x^2-\frac{2x^3}{3}-6x\right]_1^3$ $=(36-18-18)-\left(4-\frac{2}{3}-6\right)=0-\left(-2\frac{2}{3}\right)$ $=2\frac{2}{3}$	M1 A1 DM1 A1 **4**	Integrate All terms; condone one slip Substitute and subtract (even if limits wrong) M1 integrate A1 DM1 sub and sub A1
9	(i)	AB = $\sqrt{(5--1)^2+(8-1)^2}=\sqrt{85}$ AC = $\sqrt{(8--1)^2+(3-1)^2}=\sqrt{85}$	M1 A1 **2**	For sight of Pythagoras used at least once
	(ii)	M = $\left(\frac{5+8}{2},\frac{8+3}{2}\right)=\left(\frac{13}{2},\frac{11}{2}\right)$	B1 **1**	
	(iii)	Grad BC = $\frac{8-3}{5-8}=-\frac{5}{3}$ Grad AM = $\frac{11/2-1}{13/2+1}=\frac{9/2}{15/2}=\frac{9}{15}=\frac{3}{5}$ $\Rightarrow m_1.m_2=-\frac{5}{3}.\frac{3}{5}=-1$ Allow a geometric argument with reference to M being midpoint and the triangle isosceles.	E1 B1 **2**	Both gradients; AM ft from their M Both and demonstration
	(iv)	$y-1=\frac{3}{5}(x+1)$ $\Rightarrow 5y=3x+8$	M1 A1 **2**	Must use (−1, 1) or their M and their g

10	(i)	 N.B. -1 no scales	B1 E1 B1 E1 B1 **5**	One line Shading 2^{nd} line Shading Other two lines and shading
	(ii)	Maximum value on *y*-axis (0, 4) giving 12	B1 B1 **2**	Allow B2 for 12

Section B

11	(a)(i)	$x^3-3x^2-4x=0$ $\Rightarrow x\left(x^2-3x-4\right)=0$ $\Rightarrow x(x-4)(x+1)=0$ $\Rightarrow x=0,-1,4$ S.C. just answers B2	M1 A1 A1 A1 **4**	Accept any valid method
	(ii)	 Must have points on axes	B1 **1**	
	(b)(i)	Remainder theorem or long division G(–1) = 12	M1 A1 **2**	For sub –1
	(ii)	g(2) = 0	B1 **1**	For sub x = 2
	(iii)	By continued trial or by division and quadratic factorisation $g(3)=0,\ g(-2)=0$ $\Rightarrow x=2,3,-2$ S.C. just answers B2 Alternatively: By division by $(x-2)$ and quadratic factorisation M1 $(x-2)(x^2-x-6)=0$ A1 $\Rightarrow (x-2)(x+2)(x-3)=0$ A1 $\Rightarrow x = 2, -2, 3.$ A1	M1 A1 A1 A1 **4**	3 –2 Final answer

12	**(i)**	$\text{P(All males)} = \left(\frac{9}{20}\right)^8 = 0.00168$	M1 A1 **2**	
	(ii)	$\text{P(5 females)} = {}^8C_5\left(\frac{9}{20}\right)^3\left(\frac{11}{20}\right)^5$ $= 0.2568 \approx 0.257$	M1 M1 A1 A1 **4**	powers coefficient 56 (could be implied)
	(iii)	$\text{P(full-time)} = \frac{23}{40} \quad \left(\text{Or P(PT)} = \frac{17}{40}\right)$ $\text{P(at least two part-time)} = 1 - \text{P(all FT)} - \text{P(7FT,1PT)}$ $= 1 - \left(\frac{23}{40}\right)^8 - 8\left(\frac{23}{40}\right)^7\left(\frac{17}{40}\right)$ $= 1 - 0.0119 - 0.0706 = 0.917$ Alternatively: Add 7 terms M1 $28\left(\frac{23}{40}\right)^6\left(\frac{17}{40}\right)^2 + \ldots\ldots\left(\frac{17}{40}\right)^8$ A1 powers A1 Coeffs $= 0.917$ A1 Ans S.C. Read "At least two" as "exactly two" $28\left(\frac{23}{40}\right)^6\left(\frac{17}{40}\right)^2 = 28 \times 0.00653 = 0.1828$ B1	M1 A1 M1 A1 A1 A1 **6**	probability 1–2correct terms Powers coefficient Ans

13	(i)	Pythagoras: $OM^2 = 37^2 - 12^2 \Rightarrow OM = 35$ $CM^2 = 20^2 - 12^2 \Rightarrow CM = 16$	M1 A1 A1 **3**	Correct use of Pythagoras for at least one
	(ii)	Use cosine rule on triangle OCM $\Rightarrow \cos C = \dfrac{16^2 + 40^2 - 35^2}{2 \times 16 \times 40} \Rightarrow C = 60.5^0$	M1 M1 A1 A1 **4**	Correct angle Correct use of cosine formula Ans
	(iii)	Sight of attempt to find base area Area = $\frac{1}{2} \times 16 \times 24 = 192$ Sight of attempt to find height $h = 40\sin 60.5 = 34.8$ $\Rightarrow$ Volume = $\frac{1}{3} \times 192 \times 34.8 = 2228 \;\approx 2230\text{cm}^3$	M1 A1 M1 A1 A1 **5**	 Can be implied Can be implied

14	(i)	Apply Pythagoras to both triangles: $x^2 = y^2 + 4$ $(x + 0.95)^2 = (y + 1.05)^2 + 4$	 B1 B1 **2**	
	(ii)	Subtract: $2\times0.95x + 0.95^2 = 2\times1.05y + 1.05^2$ $\Rightarrow 2.1y = 1.9x - (1.05^2 - 0.95^2)$ $\Rightarrow 2.1y = 1.9x - 0.2$ Alternatively: Multiply out one of the brackets B1 Substitute for y^2 M1 Correct result A1	M1 A1 A1 **3**	
	(iii)	Substitute for y: $x^2 = \left(\frac{1.9x - 0.2}{2.1}\right)^2 + 4$ $\Rightarrow 2.1^2x^2 = 1.9^2x^2 - 2\times0.2\times1.9x + 0.2^2 + 4\times2.1^2$ $\Rightarrow 0.8x^2 + 0.76x - 17.68 = 0$ $\Rightarrow x = \frac{-0.76 \pm \sqrt{0.76^2 + 4\times0.8\times17.68}}{1.6} = \frac{-0.76 + 7.56}{1.6} = 4.25$ Substitute : $y = \left(\frac{1.9x - 0.2}{2.1}\right) = 3.75$ Withhold last mark if more than one answer given The quadratic in y is $20y^2 + 21y - 360 = 0$ Integer coefficients for x equation gives $20x^2 + 19x - 442 = 0$	M1 M1 A1 DM1 A1 DM1 F1 **7**	Get y as subject Sub expression for y Correct quadratic Solve Ignore other root

FSMQ Advanced Additional Mathematics 6993
June 2007 Assessment Session

Unit Threshold Marks

Unit	Maximum Mark	A	B	C	D	E	U
6993	100	70	60	50	40	30	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
6993	28.8	38.6	48.1	57.5	66.8	100	5500

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OCR
RECOGNISING ACHIEVEMENT

FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

6993/01

ADDITIONAL MATHEMATICS

FRIDAY 6 JUNE 2008

Afternoon
Time: 2 hours

Additional materials: Answer Booklet (16 pages)
Graph paper

You are not allowed a formulae booklet in this paper.

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 100.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **7** printed pages and **1** blank page.

Section A

1 A driver of a car, initially moving at $30\,\mathrm{m\,s^{-1}}$, applies the brakes so that the car comes to rest with constant deceleration in 10 seconds.

(i) Find the value of the deceleration. [2]

(ii) Find the distance travelled in this time. [2]

2 The points A and B have coordinates (0, 8) and (6, 0) respectively.

(i) Find the equation of the line AB. [3]

(ii) Find the equation of the line perpendicular to AB through its midpoint. [4]

3 Find the points of intersection of the line $y = 5x + 13$ with the circle $x^2 + y^2 = 13$. [5]

4 Glass marbles are produced in two colours, red and green, in the proportion 7 : 3 respectively. From a large stock of the marbles, 5 are taken at random.

Find the probability that

(i) all 5 are red, [2]

(ii) exactly 3 are red. [3]

5 **(i)** Use calculus to find the stationary points on the curve $y = x^3 - 3x + 1$, identifying which is a maximum and which is a minimum. [6]

(ii) Sketch the curve. [1]

6 A speedboat accelerates from rest so that t seconds after starting its velocity, in $\mathrm{m\,s^{-1}}$, is given by the formula $v = 0.36t^2 - 0.024t^3$.

(i) Find the acceleration at time t. [3]

(ii) Find the distance travelled in the first 10 seconds. [4]

7 A pyramid stands on a horizontal triangular base, ABC, as shown in Fig. 7.
The angles CAB and ABC are $50°$ and $60°$ respectively.
The vertex, V, is directly above C with VC = 10 m.
The angle which the edge VA makes with the vertical is $40°$.

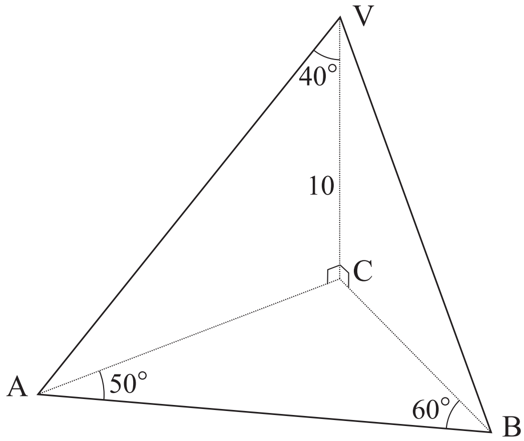

Fig. 7

(i) Calculate AC. [2]

(ii) Hence calculate AB. [4]

8 It is required to solve the equation $2\cos^2 x = 5\sin x - 1$.

(i) Show that this equation may be written as $2\sin^2 x + 5\sin x - 3 = 0$. [2]

(ii) Hence solve the equation $2\cos^2 x = 5\sin x - 1$ for values of x in the range $0° \leqslant x \leqslant 360°$. [4]

9 The cubic equation $x^3 + ax^2 + bx - 26 = 0$ has 3 positive, distinct, integer roots.

Find the values of a and b. [5]

Section B

10 Simon and Gavin drive a distance of 140 km along a motorway, both at constant speed. Simon drives at 5 km per hour faster than Gavin.

Let Gavin's speed be v km per hour.

(i) Write down expressions in terms of v for the times, in hours, taken by Gavin and Simon. [2]

Simon completes the journey in 15 minutes less than Gavin.

(ii) Explain why $\frac{140}{v} - \frac{140}{v+5} = \frac{1}{4}$ and show that this equation reduces to the equation

$$v^2 + 5v - 2800 = 0.$$ [5]

(iii) Solve this equation to find v and hence find the times taken by Simon and Gavin. Give your answers correct to the nearest minute. [5]

11 The side of a fairground slide is in the shaded shape as shown in Fig. 11. Units are metres.

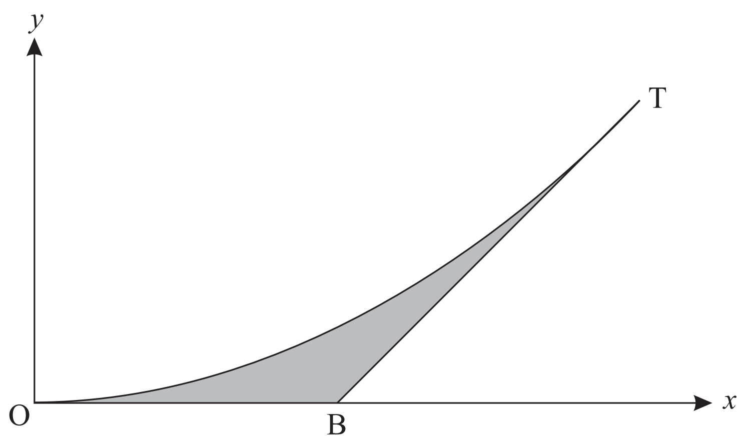

Fig. 11

The curve has equation $y = \lambda x^2$.

T has coordinates (4, 2). The line BT is a tangent to the curve at T. It meets the x-axis at the point B.

(i) Find the value of λ. [1]

(ii) Find the equation of the tangent BT and hence find the coordinates of the point B. [6]

(iii) Find the area of the shaded portion of the graph. [5]

12 A furniture manufacturer produces tables and chairs.

In each week the following constraints apply.

- There are 24 workers, each working for 40 hours (i.e. there are 960 worker-hours available).
- There is a maximum of £1800 available for the purchase of materials.
- Each table requires £30 worth of materials and 12 worker-hours.
- Each chair requires £10 worth of materials and 6 worker-hours.
- It is necessary to make at least 3 times as many chairs as tables.

Let x be the number of tables produced each week and y be the number of chairs produced each week.

(i) Show that the worker-hour constraint reduces to the inequality $2x + y \leqslant 160$. [2]

(ii) Find the inequality relating to the cost of materials constraint and the inequality relating to the numbers of tables and chairs. [3]

(iii) Plot these three inequalities on a graph, using 1 cm to represent 10 tables on the x-axis and 1 cm to represent 10 chairs on the y-axis. Indicate the region for which these inequalities hold. You should shade the region which is **not** required. [4]

When finished, each table is sold for a profit of £20 and each chair is sold for a profit of £5.

(iv) The manufacturer wishes to maximise the profit. Explain why the objective function is given by $P = 20x + 5y$. [1]

(v) Find the number of tables and chairs that should be made in order to maximise the profit. [2]

[Question 13 is printed overleaf.]

13 In the triangle shown in Fig. 13, M is the midpoint of BC.

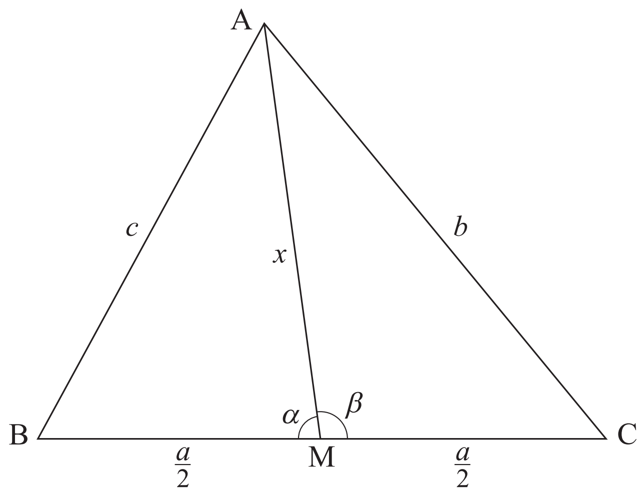

Fig. 13

(i) Explain why $\cos\alpha = -\cos\beta$. [2]

(ii) Using the cosine rule in the triangle BMA, show that

$$\cos\alpha = \frac{4x^2 + a^2 - 4c^2}{4ax}.$$ [2]

(iii) Find a similar expression for $\cos\beta$. [1]

(iv) Using the results in parts **(i)**, **(ii)** and **(iii)**, show that $4x^2 + a^2 = 2(c^2 + b^2)$. [5]

(v) A triangular lawn has sides 46 m, 29 m and 27 m. Find the distance from the midpoint of the longest side to the opposite corner. [2]

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RECOGNISING ACHIEVEMENT

FSMQ

Additional Mathematics

ADVANCED FSMQ **6993**

Mark Scheme for the Unit

June 2008

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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Additional Mathematics FSMQ (6993)

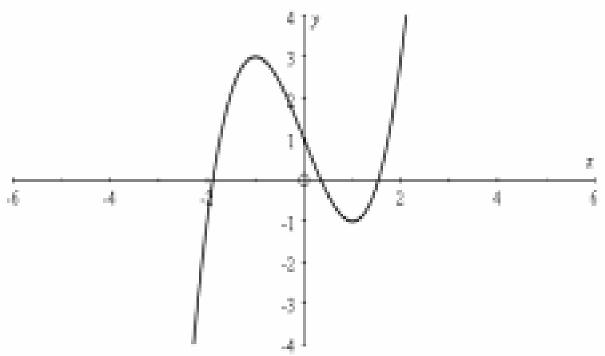
MARK SCHEME FOR THE UNIT

Unit/Content **Page**

6993 Additional Mathematics

Section A

Q.		Answer	Marks	Notes
1	(i)	$v = u + at$ with $v = 0, u = 30, t = 10$ $\Rightarrow 10a = -30$ $\Rightarrow a = -3$ Deceleration is 3 ms^{-2}	M1 A1 **2**	Must be used $a = 3$ or decel $= -3$ are wrong
	(ii)	E.g. $v^2 = u^2 + 2as$ with $v = 0, u = 30, a = -3$ $\Rightarrow 6s = 900$ $\Rightarrow s = 150$ Distance is 150 m Alternatives: $s = \left(\frac{u+v}{2}\right)t$ with $v = 0, u = 30, t = 10$ $\Rightarrow s = 15 \times 10 = 150$ Or: $s = ut + \frac{1}{2}at^2$ with $u = 30, t = 10, a = -3$ $\Rightarrow s = 300 - 150 = 150$ Or: $s = vt - \frac{1}{2}at^2$ with $v = 0, t = 10, a = -3$ $\Rightarrow s = 0 - (-150) = 150$	M1 A1 **2**	Allow alternatives
2	(i)	$\frac{x}{6} + \frac{y}{8} = 1$ $\Rightarrow 4x + 3y = 24$ Any correct equation will do. Usual answer $y = -\frac{4}{3}x + 8$ **SC.** Omission of $y =$: give M1 A0	B1 soi M1 A1 isw **3**	Gradient Any valid method In form $ax + by = c$ N.B. Drawing of graph is 0.
	(ii)	Midpoint is (3, 4) Gradient is $\frac{3}{4}$ $\Rightarrow$ equation is $y - 4 = \frac{3}{4}(x - 3)$ $\Rightarrow 4y = 3x + 7$ **SC.** Omission of $y =$: give M1 A0	B1 soi E1 M1 A1 **4**	 -ve reciprocal of their gradient Use *their* gradient plus *their* midpoint In form $ax + by = c$ N.B. Drawing of graph is 0.

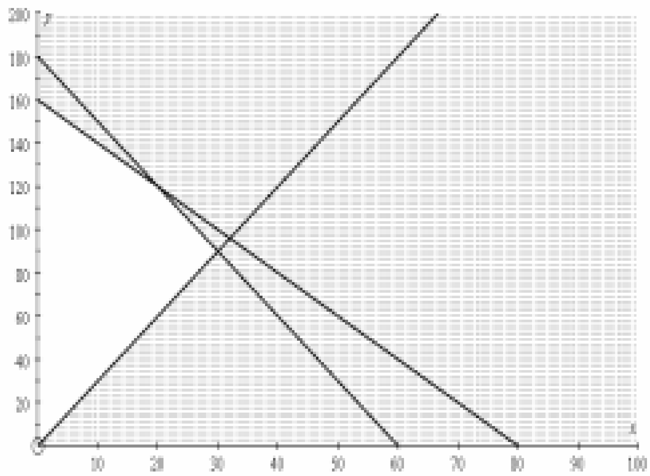
Q.		Answer	Marks	Notes
3		$x^2+(5x+13)^2=13$ $\Rightarrow x^2+25x^2+130x+169-13=0$ $\Rightarrow 26x^2+130x+156=0$ $\Rightarrow x^2+5x+6=0$ $\Rightarrow (x+2)(x+3)=0 \Rightarrow x=-2,-3$ $\Rightarrow y=3,-2$ $\Rightarrow$ Points of intersection $(-2,3),(-3,-2)$ **SC:** For each pair obtained from accurate graph or table of values, or trial, B1	M1 A1 soi M1 A1 A1 **5**	Attempt at substitution. Expansion of $(5x + 13)^2$ Solve 3 term quadratic Either both x or one pair Either both y or other pair
4	(i)	$\left(\frac{7}{10}\right)^5 \approx 0.168$	B1 soi B1 **2**	p **and** power Ans
	(ii)	$\binom{5}{3}\left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^2 \approx 0.3087$ *Allow 3, 4 or 5 sig figs in both parts* *Apply tmsf or tfsf otherwise.* [Box: 0 if more than one term]	B1 soi B1 B1 **3**	coeff powers mult (p correct) ans
5	(i)	$y=x^3-3x+1 \Rightarrow \frac{dy}{dx}=3x^2-3$ $\frac{dy}{dx}=0$ when $x=\pm1$, giving $(1,-1)$ and $(-1,3)$ $\frac{d^2y}{dx^2}=6x$; when $x=1, \frac{d^2y}{dx^2}>0$ giving minimum at $x=1$ when $x=-1, \frac{d^2y}{dx^2}<0$ giving maximum at $x=-1$ Any alternative method OK.	B1 M1 A1 A1 M1 A1 **6**	Correct derivative Setting their derivative = 0 Both x or one pair Both y or other pair *(y values could be seen in (ii))* Identify one turning point Both correct
	(ii)	 Curve to be consistent in (i)	E1 **1**	General shape including axes and turning points At *their* x values. (but don't worry about intercepts on the axes.) This *does* require a scale on the x axis.

Q.		Answer	Marks	Notes
6	(i)	$a=\frac{dv}{dt}=0.72t-0.072t^2$	M1 A1 A1 **3**	Diffn Each term
	(ii)	$s=\int_0^{10}\left(0.36t^2-0.024t^3\right)dt=\left[0.12t^3-0.006t^4\right]_0^{10}$ $=120-60=60\text{ m}$ N.B. Watch $s=\left(\frac{0+12}{2}\right)10=60$	M1 A1 M1 A1 **4**	Int the given fn Both terms Deal with def.int
7	(i)	$\frac{AC}{VC}=\tan 40\Rightarrow AC=10\tan 40=8.39\text{ m}$ **Alt forms for AC acceptable.** i.e. $AC=\frac{10\sin 40}{\sin 50}=\frac{10}{\tan 50}$	B1 B1 **2**	Tan function Correct
	(ii)	Angle $C=180-50-60=70$ $\Rightarrow\frac{AB}{\sin C}=\frac{AC}{\sin B}$ $\Rightarrow AB=8.39\times\frac{\sin 70}{\sin 60}=9.10\text{ m}$	B1 M1 F1 A1 **4**	To find AB Must be 3 s.f.
8	(i)	$2(1-\sin^2 x)=5\sin x-1$ $\Rightarrow 2\sin^2 x+5\sin x-3=0$	M1 A1 **2**	Use of pythag.to change $\cos^2$ **All working - answer given**
	(ii)	$(2\sin x-1)(\sin x+3)=0$ $\Rightarrow\sin x=\frac{1}{2}$ $\Rightarrow x=30^0,150^0$ **SC.** $\sin x=-\frac{1}{2}\Rightarrow x=210,330$ M1 A0 A0 F1	M1 A1 A1 F1 **4**	Solve quad in sin x or s etc ½ seen 30 seen 180 – ans (only one extra angle)
9		3 roots are 1, 2, 13 – allow ±1, ±2, ±13 Equation is $(x-1)(x-2)(x-13)=0$ Giving $x^3-16x^2+41x-26=0$ i.e. $a=-16$, $b=41$ (Can be seen in cubic. **Alternative method.** $f(1)=0\Rightarrow a+b=25$ B1 $f(2)=0\Rightarrow 4a+2b=18$ B1 Solve to give a and b M1 A1, A1	B1 soi B1 M1 A1 A1 isw **5**	 Factor form. Condone no = 0 Expand to give cubic

Section B

Q.		Answer	Marks	Notes
10	**(i)**	$\frac{140}{v}$, $\frac{140}{v+5}$	B1 B1 **2**	
	(ii)	Gavin's time minus Simon's time is 15 mins = $\frac{1}{4}$ hr $\Rightarrow \frac{140}{v} - \frac{140}{v+5} = \frac{1}{4}$ $\Rightarrow 4\left(140(v+5) - 140v\right) = v(v+5)$ $\Rightarrow 2800 = v(v+5) \Rightarrow v^2 + 5v - 2800 = 0$	B1 B1 M1 A1 soi A1 **5**	¼ hr Subtract Clear fractions 700
	(iii)	$v = \frac{-5 \pm \sqrt{25 + 4 \times 2800}}{2} \approx 50.47$ or 50.5 $\Rightarrow$ Gavin: 2.77 hrs, Simon 2.52 hrs $\Rightarrow$ Gavin takes 2 hrs 46 mins (166 mins) Simon takes 2 hrs 31 mins (151 mins) **SC** For $v = 50 \Rightarrow 168, 153$ give full marks but -1 tfsf	M1 A1 M1 A1 F1 **5**	Solve in decimals (ignore anything else) Convert (only one needs to be seen) Or give B1 for both in decimals This is for one 15 less than the other

Q.		Answer	Marks	Notes
11	**(i)**	$2 = 16\lambda \Rightarrow \lambda = \frac{1}{8}$	B1 **1**	
	(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{8}.2x = \frac{x}{4}$ When $x = 4, \frac{\mathrm{d}y}{\mathrm{d}x} = 1$ $\Rightarrow$ Tangent at T is $y - 2 = 1(x - 4)$ $\Rightarrow y = x - 2$ When $y = 0, x = 2$ So B is (2, 0)	E1 M1 A1 DM1 A1 A1 **6**	Correct derivative from their λ or leaving it in Sub $x = 4$ (numeric gradient to give tangent)
	(iii)	Area under curve = $\int_0^4 \frac{x^2}{8}\mathrm{d}x = \left[\frac{x^3}{24}\right]_0^4$ Area of triangle = 2 Shaded area = $\left[\frac{x^3}{24}\right]_0^4 - 2 = 2\frac{2}{3} - 2 = \frac{2}{3}$ **N.B.** Area under (curve − line) from 0 to 4 M1 A1 only	M1 A1 B1 M1 A1 **5**	Int. Function Sub limits for int and subtract triangle

Q.		Answer	Marks	Notes
12	(i)	Worker hours for tables = $12x$ Worker hours for chairs = $6y$ $\Rightarrow 12x + 6y \leq 24 \times 40 = 960 \Rightarrow 2x + y \leq 160$	M1 A1 **2**	Must see $12x$ and $6y$
	(ii)	$30x + 10y \leq 1800$ $(\Rightarrow 3x + y \leq 180)$ $y \geq 3x$	M1 A1 B1 **3**	 Does not have to be simplified
	(iii)	 N.B. Intercepts on axis must be seen N.B. Ignore < instead of ≤	B1 B1 E1 E1 **4**	Each line For $y \geq 3x$ Must be a region including the y axis as boundary
	(iv)	We wish to maximise the profit. Profit per table = 20, profit per chair = 5 i.e. $P = 20x + 5y$	B1 **1**	Something that connects 20 with x
	(v)	Greatest profit will occur where the lines $y = 3x$ and $3x + y = 180$ intersect. This is at (30, 90). Allow even if shading for $y \geq 3x$ is wrong. **SC:** Trying all corners without the corect answers B1 **SC:** Drawing an O.F. line without the right answer B1	B1 B1 **2**	30 ± 2 90 ± 2 But answers must be integers.

13	**(i)**	Angles on straight line means $\alpha = 180 - \beta$ And $\cos(180 - \beta) = -\cos\beta$	B1 B1 **2**	Must make reference to the figure of the question
	(ii)	$\cos\alpha = \dfrac{x^2 + \left(\frac{a}{2}\right)^2 - c^2}{2.\left(\frac{a}{2}\right)x}$ $= \dfrac{x^2 + \frac{1}{4}a^2 - c^2}{ax} = \dfrac{4x^2 + a^2 - 4c^2}{4ax}$	M1 A1 **2**	Correct cosine formula. Condone missing brackets.
	(iii)	$\cos\beta = \dfrac{4x^2 + a^2 - 4b^2}{4ax}$ N.B. also $-\dfrac{4x^2 + a^2 - 4c^2}{4ax}$	B1 **1**	
	(iv)	$\dfrac{4x^2 + a^2 - 4b^2}{4ax} = -\dfrac{4x^2 + a^2 - 4c^2}{4ax}$ $\Rightarrow 4x^2 + a^2 - 4b^2 = -\left(4x^2 + a^2 - 4c^2\right)$ $\Rightarrow 4x^2 + a^2 - 4b^2 = -4x^2 - a^2 + 4c^2$ $\Rightarrow 8x^2 + 2a^2 = 4\left(b^2 + c^2\right)$ $\Rightarrow 4x^2 + a^2 = 2\left(b^2 + c^2\right)$	M1 M1 A1 M1 A1 **5**	Use of (i), (ii) and (iii) Clear fractions Simplify
	(v)	$a = 46,\ b = 29,\ c = 27$ gives $4x^2 + 46^2 = 2(29^2 + 27^2)$ gives $x^2 = 256$ i.e. $x = 16$ **S.C.** Use of cosine formula in large triangle to get an angle (C = 36.2, B = 33.4) Then use of cosine formula in small triangle to get $x = 16$ M1, A1 only if the answer is 16. **SC:** Scale drawing gets 0.	M1 A1 **2**	Can be substituted in any order

Grade Thresholds

FSMQ Advanced Mathematics 6993

June 2008 Assessment Series

Unit Threshold Marks

Unit	Maximum Mark	A	B	C	D	E	U
6993	100	68	58	48	38	29	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
6993	26.4	36.7	46.5	56.0	64.7	100	7261

Statistics are correct at the time of publication

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FREE-STANDING MATHEMATICS QUALIFICATION
ADVANCED LEVEL

ADDITIONAL MATHEMATICS 6993

Candidates answer on the Answer Booklet

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- 16 page Answer Booklet
- Graph paper

Other Materials Required:
None

Friday 5 June 2009
Afternoon

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

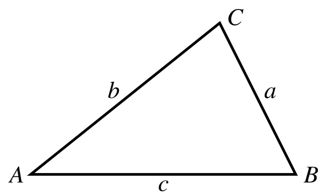
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- This document consists of **8** pages. Any blank pages are indicated.

Formulae Sheet: 6993 Additional Mathematics

In any triangle *ABC*

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$



Binomial expansion

When n is a positive integer

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Section A

1 The angle θ is greater than $90°$ and less than $360°$ and $\cos\theta = \frac{2}{3}$. Find the exact value of $\tan\theta$. **[3]**

2 Find the equation of the normal to the curve $y = x^3 + 5x - 7$ at the point $(1, -1)$. **[5]**

3 A is the point $(1, 5)$ and C is the point $(3, p)$.

(i) Find the equation of the line through A which is parallel to the line $2x + 5y = 7$. **[2]**

(ii) This line also passes through the point C. Find the value of p. **[2]**

4 AB is a diameter of a circle, where A is $(1, 1)$ and B is $(5, 3)$.

Find

(i) the exact length of AB, **[2]**

(ii) the coordinates of the midpoint of AB, **[1]**

(iii) the equation of the circle. **[3]**

5 Parcels slide down a ramp. Due to resistance the deceleration is $0.25\,\mathrm{m\,s^{-2}}$.

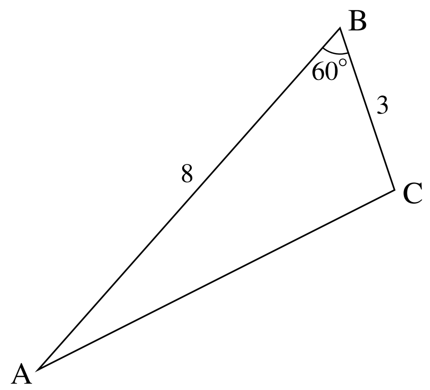
(i) One parcel is given an initial velocity of $2\,\mathrm{m\,s^{-1}}$. Find the distance travelled before the parcel comes to rest. **[3]**

(ii) A second parcel is given an initial velocity of $3\,\mathrm{m\,s^{-1}}$ and takes 4 seconds to reach the bottom of the ramp. Find the length of the ramp. **[3]**

6 The gradient function of a curve is given by $\frac{dy}{dx} = 1 - 4x + 3x^2$.

Find the equation of the curve given that it passes through the point $(2, 6)$. **[4]**

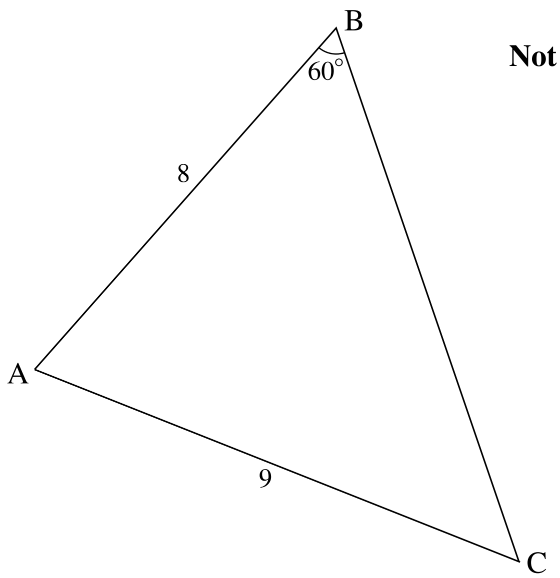
7 The course of a cross-country race is in the shape of a triangle ABC.
AB = 8 km, BC = 3 km and angle ABC = 60°.



Not to scale

(i) Calculate the distance AC and hence the total length of the course. **[4]**

(ii) The organisers extend the course so that AC = 9 km.



Not to scale

Calculate the angle BCA. **[3]**

8 Calculate the x-coordinates of the points of intersection of the line $y = 2x + 11$ and the curve $y = x^2 - x + 5$. Give your answers correct to 2 decimal places. **[5]**

9 A car accelerates from rest. At time t seconds, its acceleration is given by $a = 4 - 0.2t \text{ m s}^{-2}$ until $t = 20$.

(i) Find the velocity after 5 seconds. **[3]**

(ii) What is happening to the velocity at $t = 20$? **[1]**

(iii) Find the distance travelled in the first 20 seconds. **[3]**

10 **(i)** Illustrate on one graph the following three inequalities.

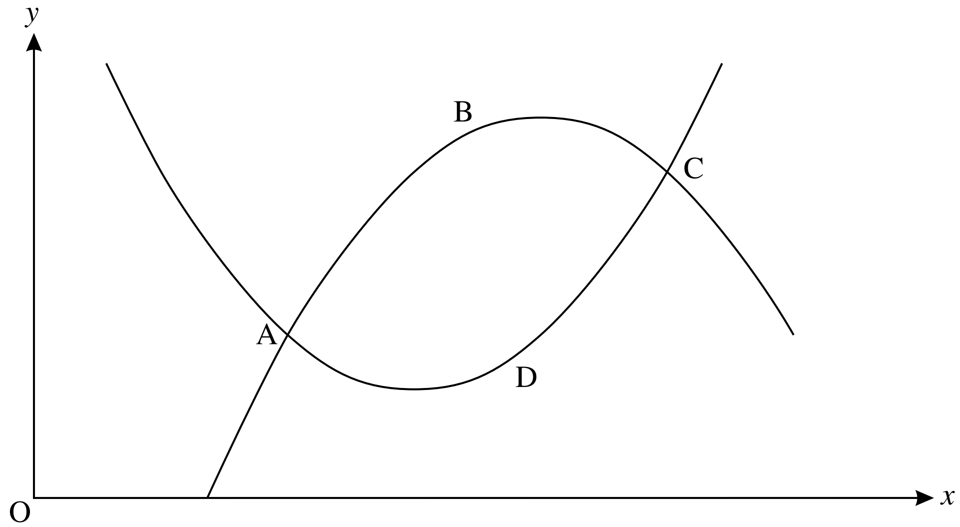
$$y \geqslant x - 1$$
$$x \geqslant 2$$
$$2x + y \geqslant 8$$

Draw suitable boundaries and shade areas that are **excluded**. **[4]**

(ii) Write down the minimum value of y in this region. **[1]**

Section B

11 The shape ABCD below represents a leaf.
The curve ABC has equation $y = -x^2 + 8x - 9$.
The curve ADC has equation $y = x^2 - 6x + 11$.

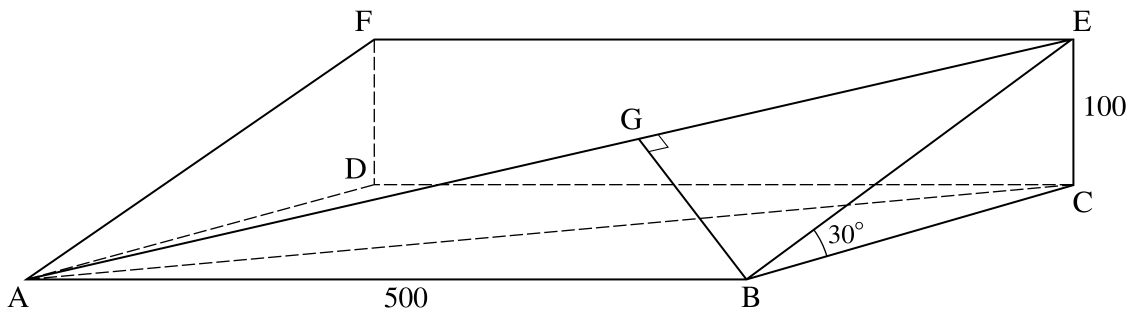


(i) Find algebraically the coordinates of A and C, the points where the curves intersect. **[5]**

(ii) Find the area of the leaf. **[7]**

12 The diagram shows a rectangle ABEF on a plane hillside which slopes at an angle of 30° to the horizontal. ABCD is a horizontal rectangle. E and F are 100 m vertically above C and D respectively. AB = DC = FE = 500 m.

AE is a straight path.
From B there is a straight path which runs at right angles to AE, meeting it at G.



(i) Find the distance BE. **[3]**

(ii) Find the angle that the path AE makes with the horizontal. **[4]**

(iii) Find the area of the triangle ABE.

Hence find the length BG. **[5]**

13 In a supermarket chain there are a large number of employees, of whom 40% are male.

(a) One employee is chosen to undergo training.
What assumption is made if 0.4 is taken to be the probability that this employee is male? **[1]**

(b) 6 employees are chosen at random to undergo training.

(i) Show that P(all 6 chosen are female) = 0.0467, correct to 4 decimal places. **[2]**

Find the probability that

(ii) 3 are male and 3 are female, **[4]**

(iii) there are more females than males chosen. **[5]**

14 (a) (i) On the same graph, draw sketches of the curve $y = x^3$ and the line $y = 3 - 2x$. **[2]**

(ii) Use your sketch to explain why the equation $x^3 + 2x - 3 = 0$ has only one root. **[1]**

(b) (i) Show by differentiation that there are no stationary points on the curve $y = x^3 + 3x - 4$. **[3]**

(ii) Hence explain why the equation $x^3 + 3x - 4 = 0$ has only one root. **[1]**

(c) (i) Use the factor theorem to find an integer root of the equation $x^3 + x - 10 = 0$. **[1]**

(ii) Write the equation $x^3 + x - 10 = 0$ in the form $(x - a)(x^2 + px + q) = 0$ where a, p and q are values to be determined. **[2]**

(iii) By considering the quadratic equation $x^2 + px + q = 0$ found in part **(ii)**, show that the cubic equation $x^3 + x - 10 = 0$ has only one root. **[1]**

(d) You are given that r and s are positive numbers. What do the results in parts **(a)**, **(b)** and **(c)** suggest about the equation $x^3 + rx - s = 0$? **[1]**

OCR
RECOGNISING ACHIEVEMENT

FSMQ

Additional Mathematics

FSMQ **6993**

Mark Schemes for the Units

June 2009

6993/MS/R/09

Oxford Cambridge and RSA Examinations

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CONTENTS

Foundations of Advanced Mathematics FSMQ (6993)

MARKSCHEME ON THE UNIT

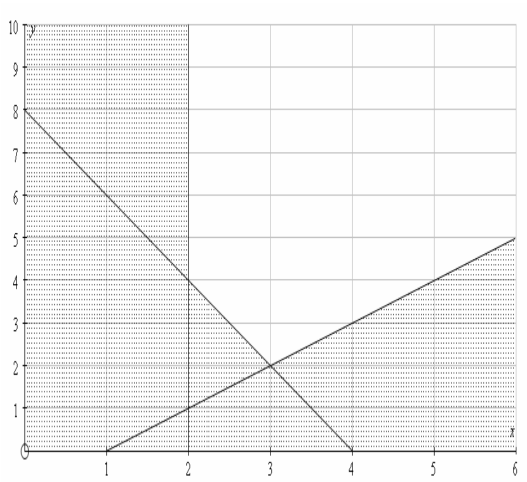
Additional Mathematics – 6993

Section A

1		Pythagoras for third value: $c=\sqrt{5}$	M1 A1	Using any means to find $\sqrt{5}$
		$\Rightarrow \tan\theta = -\frac{\sqrt{5}}{2}$	A1 **3**	Includes negative sign.
		Alt: $\sin^2\theta = 1-\cos^2\theta = 1-\frac{4}{9}=\frac{5}{9}$	M1	Use of Pythagoras
		$\Rightarrow \sin\theta = \frac{1}{3}\sqrt{5}$	A1	Sin θ
		$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{5}/3}{2/3} = -\frac{\sqrt{5}}{2}$	A1	Includes negative sign
		SC: Allow B1 for $\tan\theta = -1.12$		
2		$\frac{dy}{dx} = 3x^2+5$	M1	Attempt at differentiation with at least one term with correct power
		$\Rightarrow$ grad tangent = 8	A1	
		$\Rightarrow$ grad normal $= -\frac{1}{8}$	F1	
		$\Rightarrow y+1 = -\frac{1}{8}(x-1)$	M1	Dep on use of their normal gradient and correct point
		$\Rightarrow 8y+8=-x+1 \Rightarrow 8y+x+7=0$	A1 **5**	Any acceptable form. Acceptable means three terms only
3	**(i)**	$2x+5y=2+25$	M1	Substitute new point to change c
		$\Rightarrow 2x+5y=27$	A1 **2**	If put in form $y = mx + c$ then $m = -0.4$
		SC: B2 from scale drawing only if absolutely correct		
	(ii)	When $x=3,\ 6+5y=27$	M1	Substituting $x = 3$ into either their equation from (i) **or** the given equation in (i)
		$\Rightarrow 5y=21 \Rightarrow y=\frac{21}{5}$		
		$\Rightarrow p=\frac{21}{5}=4.2$	F1 **2**	Answer must specifically give p
		NB $p = 0.2$ comes from using original line. Give M1 A1 for this.		

4	(i)	$AB=\sqrt{(5-1)^2+(3-1)^2}$ $=\sqrt{4^2+2^2}$ $=\sqrt{20}=2\sqrt{5}$	M1 A1 2	 isw ie ignore any approx value for root.
		NB M1 A0 for 4.47 with no sight of $\sqrt{20}$		
	(ii)	$\left(\frac{1+5}{2},\frac{1+3}{2}\right)=(3,2)$	B1 1	
	(iii)	$(x\pm a)^2+(y\pm b)^2$ with (a,b) from **(ii)** $(x-a)^2+(y-b)^2$ $=5$	M1 F1 A1 3	Use of equation Their midpoint cao for 5 isw ie ignore any incorrect algebra following a correct equation
5	(i)	$v^2=u^2+2as\Rightarrow 0=4-2\times 0.25s$ $\Rightarrow s=8$ Distance travelled = 8 m	M1 A1 A1 3	Use of right formula(e) Substitution Answer
		If t is found first then M1 for any correct equations that lead to finding s Careful also of $4=0+\frac{1}{2}s$, this could be 3 if quoted formula is right. Also of $0=4+\frac{1}{2}s\Rightarrow s=-8$ Both of these M1 for formula only		
	(ii)	$s=ut+\frac{1}{2}at^2=s=3\times 4-\frac{1}{2}\times 0.25\times 16$ $=12-2=10$ Length of ramp = 10 m	M1 A1 A1 3	
		NB Anything that uses $v=0$ is M0		
6		$\frac{dy}{dx}=1-4x+3x^2$ $\Rightarrow (y=)x-2x^2+x^3(+c)$ Through (2, 6) $\Rightarrow 6=2-8+8+c\Rightarrow c=4$ $\Rightarrow y=x-2x^2+x^3+4$	M1 A1 M1 A1 4	For integrating - increase in power of one in at least two terms Attempt to find c Must be an equation

7	(i)	$AC^2 = 8^2 + 3^2 - 2.8.3.\cos 60$ $= 73 - 24 = 49$ $\Rightarrow AC = 7$ $\Rightarrow$ Total distance = 18 km	M1 A1 A1 F1 **4**	Use of formula AC Total distance
	(ii)	$\frac{\sin \text{BCA}}{8} = \frac{\sin 60}{9}$ $\Rightarrow \sin \text{BCA} = \frac{8}{9} \times \sin 60 (= 0.7698)$ $\Rightarrow \text{BCA} = 50.3^0$	M1 A1 A1 **3**	
		Alternative Scheme: Use of cosine formula twice $\Rightarrow$ BC = 9.74… $\Rightarrow$ BCA = 50.3^0	 M1 A1 A1	
8		$2x + 11 = x^2 - x + 5$ $\Rightarrow x^2 - 3x - 6 \ (= 0)$ $\Rightarrow x = \frac{3 \pm \sqrt{9+24}}{2} = \frac{3 \pm \sqrt{33}}{2}$ $= 4.37$ or -1.37	M1 A1 M1 A1 A1 **5**	Substitute Quadratic Solve Correct substitution Both answers Ignore values for y
		Alternative Scheme 1 (relates to last 3 marks) Completion of square: $(x - 1.5)^2 = k$ $x - 1.5 = \pm\sqrt{8.25}$ $\Rightarrow x = 4.37$ or -1.37	 M1 A1 A1	 Must contain ± Must be 2 dp
		Alternative Scheme 2: Only 2 marks from last 3 Solving their quadratic by T&I Both roots	 M1 A1	
		Alternative Scheme 3. Only 4 marks Roots with no working: B2 each	 B2,2	
		Alternative Scheme 4. Only 4 marks Finding a root from the original equations = one of them Finding the second root = the other	 M1 A1 M1 A1	
		Alternative scheme 5. Eliminate x. Gives $y^2 - 28y + 163 = 0$ Gives y = 19.74 and 8.26 leading to x values	M1 A1 M1 A1 A1	Eliminate x Quadratic Solve Both y values Both x values
		NB Attempt to solve by graph - M0		

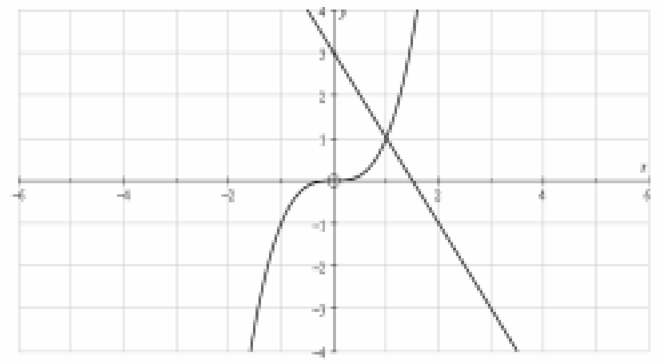
9	**(i)**	$a = 4 - 0.2t$ $\Rightarrow v = 4t - 0.1t^2$ $\Rightarrow v_5 = 20 - 2.5 = 17.5$ Velocity is 17.5 m s^{-1}	M1 A1 A1 **3**	Integrate (increase of power of one in at least one term) Ignore *c*
	(ii)	At $t = 20$, $a = 0$ ie Maximum velocity	B1 **1**	
	(iii)	$v = 4t - 0.1t^2$ $\Rightarrow s = \int_0^{20} 4t - 0.1t^2 \, dt = \left[2t^2 - 0.1\frac{t^3}{3}\right]_0^{20}$ $= 2 \times 400 - 0.1 \times \frac{8000}{3} = 533.3... = 533$ Distance travelled = 533 m	M1 A1 A1 **3**	Integrate their *v* from (i) (Increase in power of one term) Ignore *c* Allow exact answer or 3sf
10	**(i)**		B2,1 B2,1 **4**	Lines, −1 each error Shading, −1 each error Correct side of line. ft if gradient is the same sign.
	(ii)	$y = 2$	E1 **1**	ft their graph

Section B

11	**(i)**	$-x^2+8x-9=x^2-6x+11$ $\Rightarrow 2x^2-14x+20=0$ $\Rightarrow x^2-7x+10=0$ $\Rightarrow (x-5)(x-2)=0$ $\Rightarrow x=2,\ 5$ Substitute: $x=2 \Rightarrow y=4-12+11=3$ $x=5 \Rightarrow y=25-30+11=6$	M1 A1 M1 A1 A1 **5**	Equate Quadratic Solve: Factorisation needs 2 numbers to multiply to their constant Or one pair, e.g. (2,3) or (5,6)
		Alternative scheme: Completion of square: $(x-3.5)^2=k$ $x-3.5=\pm\sqrt{2.25}$ $\Rightarrow x=5 \text{ or } 2$ $\Rightarrow y=6 \text{ or } 3$	M1 A1 A1	
	(ii)	$A=\int_2^5 (y_1-y_2)\,dx=\int_2^5\left(-2x^2+14x-20\right)dx$ $=\left[-\frac{2x^3}{3}+7x^2-20x\right]_2^5$ $=\left(-\frac{2\times125}{3}+7\times25-100\right)-\left(-\frac{16}{3}+28-40\right)$ $=\left(-\frac{250}{3}+75\right)-\left(-\frac{16}{3}-12\right)=-\frac{234}{3}+87=87-78=9$	M1 A1 M1 A2 M1 A1 **7**	Int between curves ± Correct expression Integrate their function (not if they divide by 2) All terms, −1 for each error Sub into integral Answer
		Alternative scheme: $A=\int_2^5\left(-x^2+8x-9\right)dx-\int_2^5\left(x^2-6x+11\right)dx$ $=\left[-\frac{x^3}{3}+4x^2-9x\right]_2^5-\left[\frac{x^3}{3}-3x^2+11x\right]_2^5$ $=\left(\left(-\frac{125}{3}+100-45\right)-\left(-\frac{8}{3}+16-18\right)\right)$ $-\left(\left(\frac{125}{3}-75+55\right)-\left(\frac{8}{3}-12+22\right)\right)$ $=\left(13\frac{1}{3}-\left(-4\frac{2}{3}\right)\right)-\left(21\frac{2}{3}-12\frac{2}{3}\right)=18-9$ $=9$	M1 M1 A1 A1 M1 A1 A1	Subtracting 2 integrals Integrate either All terms of y_1 All terms of y_2 Substitute into either integral For 18 or 9 Final answer
		SC $A=\int\left(y_1+y_2\right)\,dx$ M1 integrate and M1 sub only		

12	(i)	$\frac{100}{BE} = \sin 30$ $\Rightarrow BE = \frac{100}{\sin 30} = 200$ m	M1 A1 A1 3	Fraction right way up Correct expression for BE Or B3 if the special triangle is noticed.
		Alternative scheme: $\frac{100}{BC} = \tan 30 \Rightarrow BC = \frac{100}{\tan 30} = 173.2$ $BE = \sqrt{100^2 + 173.2^2} = 200$	M1 A1 A1	Ratio and Pythagoras Allow not exact
	(ii)	AE by Pythagoras: $AE = \sqrt{500^2 + 200^2} = 100\sqrt{29} = 538.5...$ $\sin A = \frac{100}{538.5}$ $\Rightarrow A = 10.7^0$	M1 A1 M1 A1 4	soi
		Alternative Scheme: $BC = \sqrt{30000} \approx 173.2 \Rightarrow AC = \sqrt{280000} \approx 529.2$ $\Rightarrow A = \tan^{-1}\frac{100}{\sqrt{280000}} = 10.7^0$ NB $A = 10.9^0$ comes from $\sin^{-1}\frac{100}{\sqrt{280000}}$	M1 A1 M1 A1	
	(iii)	Area $= \frac{1}{2} \times 500 \times$ their BE $= 50\,000$ Area $= \frac{1}{2} \times BG \times$ their AE $\Rightarrow BG = \frac{2 \times \text{their area}}{\text{their AE}} = 185.7...... \approx 186$ m	M1 A1 M1 A1 A1 5	
		Alternative Scheme: Find angle A or E Then $\frac{BG}{500} = \sin A \Rightarrow BG = 186$ m ie maximum 3 marks. The answer is found, but the question says "Hence" and this is "otherwise". NB If area is attempted but not used then give M1 A1. If area is found after BG is found then do not mark it.	M1 A1 A1	

		In all parts of this question allow answers to 3sf or 4 dp		
13	**(a)**	The selection is random. *Allow anything that implies equal chance of selection*	B1 **1**	
	(b)(i)	P(all are female) = 0.6^6 (= 0.046656) = 0.0467	M1 A1 **2**	Sight of 0.6^6 Must be 3 sf
	(ii)	P(3 of each) = Bin coeff × $0.6^3 \times 0.4^3$ = 20 × $0.6^3 \times 0.4^3$ = 0.2765 or 0.276	M1 A1 A1 A1 **4**	One term with binomial coeff 20 (may be implied) Powers (may be implied)
	(iii)	P(more females than males) = 6, 0 or 5,1 or 4,2 = $0.6^6 + 6 \times 0.6^5 \times 0.4 + 15 \times 0.6^4 \times 0.4^2$ = 0.04666 + 0.1866 + 0.3110 = 0.5443 Allow 0.544, 0.545, 0.5444	M1 B1 B1 B1 A1 **5**	Add 3 terms Binomial coefficients correct in at least two terms Powers correct in at least two terms At least 2 terms correct.
		Alternative scheme: P(more females than males) = 1 – P(more males than females *or* equal numbers) = 1 – (0.4^6+6×0.4^5×0.6+15×0.4^4×0.6^2+20×0.4^3 ×0.6^3 = 1 – (0.0041 + 0.0369 + 0.1382 + 0.2765) = 0.5443	M1 B1 B1 B1 A1	Take 4 terms from 1 Binomial coeffs Powers At least 2 terms correct
		The terms are: 0.0467, 0.1866, 0.3110, 0.2765, 0.1382, 0.0369, 0.0041		
		If P(more males than females), treat as MR and –2 If p = 0.4 and q = 0.6 then MR –2 (but also 0 for (b)(i) where answer is given!)		

14	**(a)(i)**		B1 B1 **2**	Line with +ve intercepts and –ve gradient Curve Condone +ve gradient for cubic at origin. Must pass through the origin
	(ii)	Can only intersect in one point.	B1 **1**	Allow if obviously true, even if one or both are wrong
		NB Do not allow if the curve implies that there could be more than one root but the line has not been drawn long enough - eg if curve is quadratic		
	(b)(i)	$\frac{dy}{dx} = 3x^2 + 3$ Greater than 0 for all x or attempt to solve their $\frac{dy}{dx} = 0$ so no solution to $3x^2 + 3 = 0$	B1 M1 A1 **3**	Correct two terms $= 0$ No solution
	(ii)	Because the curve is always increasing can only cross the x axis in one point which is the root	B1 **1**	There must be some reference to **(b)(i)**
	(c)(i)	By trial f(2) = 0 Condone $(x - 2)$ is a factor	B1 **1**	
	(ii)	$\Rightarrow (x - 2)(x^2 + 2x + 5) = 0$	M1 A1 **2**	In long division at least x^2 must be seen
	(iii)	Discriminant "$b^2 - 4ac$" = -16 < 0 So no roots. This means that $x = 2$ is the only root.	B1 **1**	Depends on **(ii)** being correct
		NB "Quad does not factorise" is not good enough		
	(d)	The equation will only have one root (for all r and s.)	B1 **1**	Ignore extra comments even if wrong

Grade Thresholds

Additional Mathematics (6993)
June 2009 Assessment Series

Unit Threshold Marks

Unit	Maximum Mark	A	B	C	D	E	U
6993	100	73	63	53	44	35	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
6993	27.7	39.7	48.7	56.9	66.0	100	9859

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FREE-STANDING MATHEMATICS QUALIFICATION
ADVANCED LEVEL

Additional Mathematics **6993**

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 16 page Answer Booklet
- Graph paper

Other Materials Required:
None

Tuesday 15 June 2010
Morning

Duration: 2 hours

* 6 9 9 3 *

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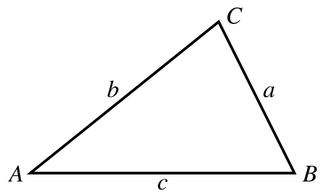


Binomial expansion

When n is a positive integer

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Section A

1 Solve the inequality $3 - x < 4(x - 1)$. **[3]**

2 Expand $(1 - x)^{12}$ in ascending powers of x up to the term in x^3, and simplify your answer. **[3]**

3 The function $f(x)$ is defined by $f(x) = x^3 - 5x^2 + 2x + 8$.

(i) Find the remainder when $f(x)$ is divided by $(x + 1)$. **[2]**

(ii) Solve the equation $f(x) = 0$. **[3]**

4 In a game 4 fair dice are thrown.

Calculate the probability that

(i) no six is thrown, **[2]**

(ii) at least 2 sixes are thrown. **[4]**

5 The curve $y = x^3 - 3x^2 - 9x + 7$ has two turning points, one of which is where $x = 3$.

(i) Find the coordinates of the other turning point and determine whether it is a maximum or minimum point. **[5]**

(ii) Sketch the curve. **[1]**

6 An aeroplane touches down at a point A on a runway, travelling at $90\,\text{m}\,\text{s}^{-1}$. It then decelerates uniformly until it reaches a speed of $6\,\text{m}\,\text{s}^{-1}$ at a point B on the runway, 2016 m from A.

(i) Find the deceleration. **[3]**

(ii) Find the time taken to travel from A to B. **[2]**

7 It is required to solve the equation $\sin\theta\cos\theta = \frac{1}{4}$.

(i) Show that $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\sin\theta\cos\theta}$. **[1]**

(ii) Hence show that the equation $\sin\theta\cos\theta = \frac{1}{4}$ is equivalent to $\tan\theta + \dfrac{1}{\tan\theta} = 4$. **[2]**

(iii) By expressing this equation as a quadratic equation in t, where $t = \tan\theta$, find the two values of θ, in the range $0° \leqslant \theta \leqslant 180°$, that satisfy the equation. **[4]**

8 A train moves between two stations, taking 5 minutes for the journey.
The velocity of the train may be modelled by the equation $v = 60(t^4 - 10t^3 + 25t^2)$ where v is measured in metres per minute and t is measured in minutes.

Calculate the distance between the two stations. **[5]**

9 The diameter of a circle is PQ, where P and Q are the points (1, 3) and (15, 1) respectively.

(i) Find the centre of the circle. **[2]**

(ii) Show that the radius of the circle is $5\sqrt{2}$. **[2]**

(iii) Hence find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. **[2]**

10 John and Paul are carrying out an experiment.

The table shows their results for x and y.

x	0	2	3	4
y	4	0	0.25	0

Paul proposes that the relationship should be modelled by $y = k(x-2)(x-4)$. This is shown in Fig. 10.

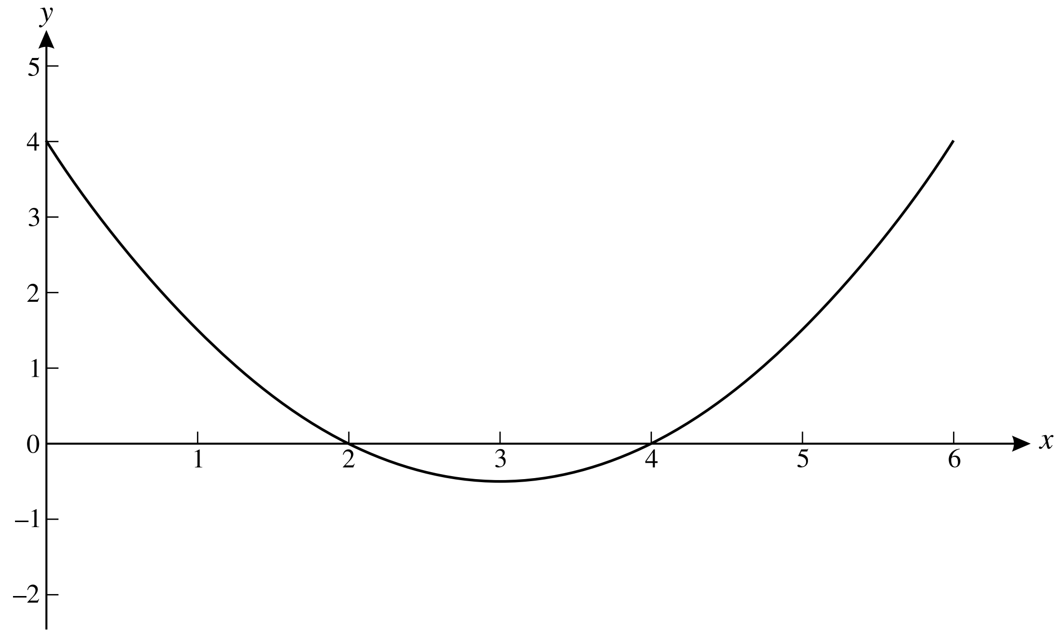


Fig. 10

(i) Find the value of k for which the points (0, 4), (2, 0) and (4, 0) satisfy this equation. **[2]**

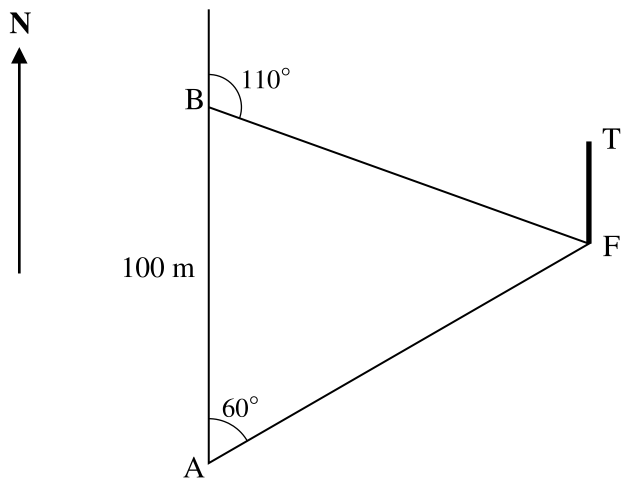
John proposes a different model, using $y = c(x-2)^2(x-4)$.

(ii) Find the value of c for which the points (0, 4), (2, 0) and (4, 0) satisfy this equation. **[2]**

(iii) Which is the better model for John and Paul's results? Give a reason for your answer. **[2]**

Section B

11 Michael is at a point A and the base of a church tower is at a point F, as shown in Fig. 11.
He measures the bearing of the tower to be 060°.
Michael walks 100 metres due North to the point B from where he measures the bearing of F to be 110°.
The triangle ABF is in the horizontal plane.



Not to scale

Fig. 11

(i) Show that AF = 122.7 m, correct to 4 significant figures, and find BF. **[5]**

Michael finds that the angle of elevation of the top of the tower, T, from A is 10°.

(ii) Find the height of the tower. **[2]**

C is the point on AB that is nearest to F.

(iii) Find CF and the angle of elevation from C to the top of the tower, correct to 1 decimal place. **[5]**

12 Fig. 12 shows the shape AOB that is to be made from card.

B is the point (5, 0) and OB is part of the curve with equation $y = 0.3x^2 - 1.5x$.

The line AB is the normal to the curve at B.

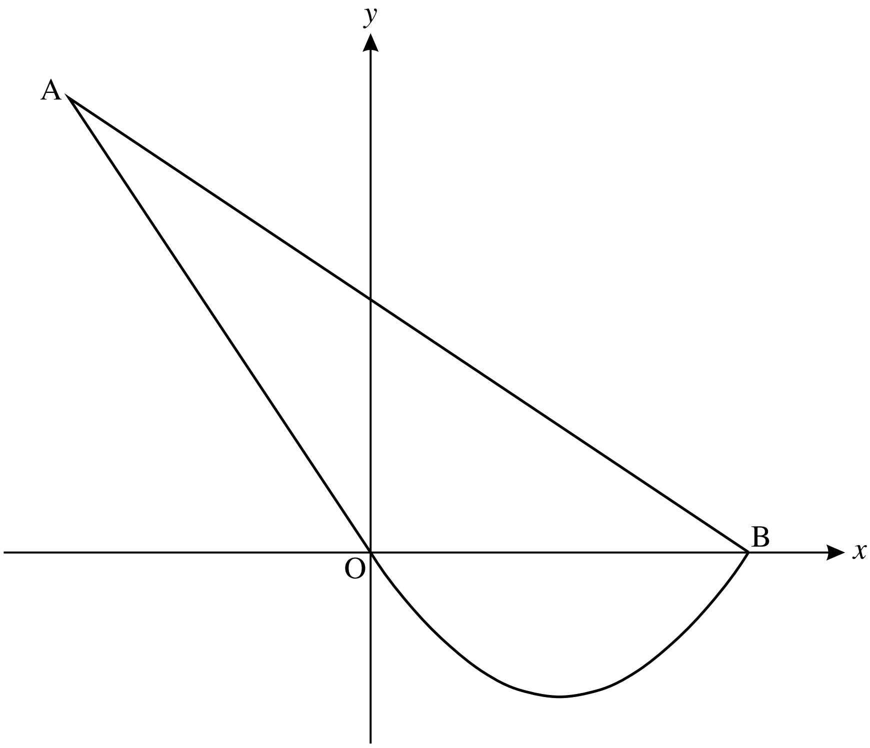


Fig. 12

(i) Find the equation of the line AB. **[4]**

The equation of the line AO is $2y + 3x = 0$.

(ii) Find the coordinates of the point A. **[3]**

(iii) Find the area of the shape AOB. **[5]**

[Questions 13 and 14 are printed overleaf.]

13 Ali and Beth make components in a factory. Ali works faster than Beth and makes 3 more components per hour. As a result he takes 2 hours less time than Beth to make 72 components.

Let t hours be the time that Ali takes to make 72 components.

(i) Write expressions for the numbers of components made per hour by Ali and by Beth. **[3]**

(ii) Hence derive the equation $3t(t+2) = 144$. **[5]**

(iii) Solve this equation to find the times that Ali and Beth take to make 72 components. **[4]**

14 A firm has to transport 1500 packages to a site. It has a number of large vans which will transport 200 packages each and a number of small vans which will transport 100 packages each.

Let x be the number of large vans and let y be the number of small vans used.

(i) Write down an inequality based on the number of packages transported. **[2]**

The firm needs to use at least as many small vans as large vans.

(ii) Write a second inequality. **[1]**

(iii) Plot these two inequalities on a graph, using 1 cm to represent one van on each axis. Indicate the region for which these inequalities hold. Shade the area that is **not** required. **[3]**

A large van costs £80 to complete the trip and a small van costs £60 to complete the trip.

(iv) Write down the objective function and hence find from your graph the number of each type of van that will minimise the cost, and work out that cost. **[4]**

(v) What choice of vans should be made to minimise the cost if the restriction about the large and small vans is removed? Work out the cost in this case. **[2]**

RECOGNISING ACHIEVEMENT

FSMQ

Advanced FSMQ

Additional Mathematics **6993**

Mark Scheme for June 2010

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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OCR - ADDITIONAL MATHEMATICS 6993
Marking instructions.

The total mark for the paper is **100.**

Marks for method are indicated by an **M**. A method that is dependent on previous work is **DM.**

Marks for accuracy are of two kinds:
- (i) **A** mark indicates correct work only and
- (ii) **F** mark indicates that a "follow through" is allowed.

If an **M** mark is not gained then nor do any of the accuracy marks associated with it.

Marks not associated with a method are denoted **B**, which should be treated as "correct only", and **E** which may be wrong because of a previous error.

Marks are not divisible except as indicated. e.g. A 2,1 means that 2 are awarded for a correct answer and 1 for an answer that is only partially correct, as agreed at the meeting of Examiners.

When the method of solution is not one that has been discussed and does not fit the existing scheme then an alternative scheme should be devised which maintains the same number of M, A, F, B and E marks. You should also bring this to the attention of the Principal Examiner.

The rubric says that the norm is for answers to be given to 3 s.f. except where indicated. Where this rubric is ignored then 1 mark should be deducted once in the paper, at the point where it is first met. This should be indicated -1, TMSF or -1TFSF. Details will be discussed at the meeting of examiners.

Misreading of a question (including the candidate's own working) should normally be penalised by the loss of the relevant accuracy mark or two marks (whichever is less); but if the question is made substantially easier then further penalties may be imposed.

Sub-marks should be shown near to the relevant work. If these are individual marks then the appropriate letter should be given. Sub-marks are given in the question paper and the mark scheme. For substantially correct solutions a number of sub-marks may be combined, even up to the total mark for the question for a totally correct question. The sum of the sub-marks are then added and ringed at the end of the question. (This means that a totally correct question has the total mark written twice - once as a "sum of sub-marks" and unringed and once ringed as the total for the question.) The total mark for the paper should be given on the front page, top right and ringed.

Work that is crossed out and not replaced should be marked. If work has been crossed out and replaced then the replacement work should be marked even if it is incorrect and the crossed out work correct.

Any notation that is understandable may be used to support your marking. In particular:
isw – ignore subsequent working
www – without wrong working
soi – seen or implied

An independent person should be used to check the summation of marks. You should add the ringed marks on the paper to check the addition and the independent checker should add the unringed marks. There is a fee paid for this checking - if you are unable to find anyone to do this work the Board and the Principal Examiner must be informed.

Please mark in red.

If examiners have any doubt about the interpretation of any instructions or if any point of difficulty arises during the marking of scripts, they should communicate with the Principal Examiner.

Section A

1		$3-x<4(x-1)$ $\Rightarrow 3-x<4x-4$ $\Rightarrow 7<5x$ $\Rightarrow x>\frac{7}{5}$	B1 B1 B1 **3**	Sight of $4x-4$ Sight of ax and b where either $a=5$ or $b=7$ oe Final answer WWW
2		$=1-\binom{12}{1}x+\binom{12}{2}x^2-\binom{12}{3}x^3$ $=1-12x+66x^2-220x^3$ *Ignore terms of higher power*	B1 B1 B1 **3**	Signs and powers 2 out of 3 coefficients worked out All coefficients and 1
3	**(i)**	Remainder is f(−1) $=-1-5-2+8=0$ *For long division x^3+x^2 in working and x^2 in quotient must be seen for M1* *Or by inspection $(x+1)(x^2+\ldots)$ for M1*	M1 A1 **2**	Or long division 0 must be seen or implied
	(ii)	$x^3-5x^2+2x+8=0$ $\Rightarrow (x+1)(x^2-6x+8)=0$ $\Rightarrow (x+1)(x-2)(x-4)=0$ $\Rightarrow x=-1,2,4$ *Allow ans with no working*	M1 DM1 A1 **3**	Factorise cubic to give $(x+1)(ax^2+bx+c)$ Solve their quadratic
		Alt: Trial to find one root: $x=2, 4$ M1, A1 $\Rightarrow x=-1, 2, 4$ A1		

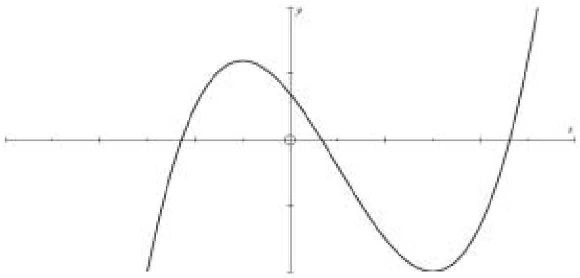
4	(i)	$\left(\frac{5}{6}\right)^4 = \frac{625}{1296} = 0.4823$	M1 A1 **2**	Either form or 0.482 isw
	(ii)	$1-\left(\frac{5}{6}\right)^4 - 4\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)$ $= 1 - \frac{625}{1296} - \frac{500}{1296} = 1 - 0.4823 - 0.3858$ $= \frac{171}{1296} = \frac{19}{144} = 0.1319$	M1 B1 B1 A1 **4**	1 − 2 terms 4 soi Powers Ans in either form or 0.132
		Alt: Add three terms M1 $6\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2 + 4\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^4$ B1 both coeffs, B1 powers $= 0.11574 + 0.01543 + 0.00077$ $= 0.1319$ A1 ans		

Q	Part	Answer	Marks	Guidance
5	(i)	$\frac{dy}{dx}=3x^2-6x-9$	M1	Diffn **and** set = 0
		$=0$ when $3x^2-6x-9=0 \Rightarrow x^2-2x-3=0$	A1	Derived fn
		$\Rightarrow (x-3)(x+1)=0 \Rightarrow x=3,-1$		
		When $x=-1, y=12$	A1	Stationary point
		$\frac{d^2y}{dx^2}=6x-6<0$ when $x=-1$ so maximum	M1 A1	To find nature of turning points
		Allow SC1 for (–1, 12) with no working	5	
		Alternative ways to demonstrate maximum at $x=-1$ Value of y	M1 A1	Allow at most one integer either side (typically, $x=-2$, 0 if turning point is correct)

− 1 −	− 1	−1 +
$y<12$	$y=12$	$y<12$

Gradient of tangent — M1, A1

− 1 −	− 1	−1 +
$\frac{dy}{dx}>0$	$\frac{dy}{dx}=0$	$\frac{dy}{dx}<0$
/	—	\

Q	Part	Answer	Marks	Guidance
	(ii)		B1 1	General shape: turning points in correct quadrants Intercept on y axis in [0,12] Does not turn back on itself.
6	(i)	$u=90, v=6, s=2016$ $\Rightarrow 6^2=90^2+2a\times 2016$ $\Rightarrow a=-\frac{90^2-6^2}{4032}=-\frac{8064}{4032}=-2 \text{ m s}^{-2}$	M1 A1 A1 3	Using correct formula Correct substitution
	(ii)	$u=90, v=6, a=-2$ $\Rightarrow 6=90-2t$ $\Rightarrow t=\frac{84}{2}=42$ secs *The two parts can be the other way round*	M1 A1 2	Using correct formula

7	**(i)**	$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$ $=\frac{1}{\sin\theta\cos\theta}$	B1 **1**	
		Alt: $\sin^2\theta+\cos^2\theta=1$ $\Rightarrow \sin\theta+\frac{\cos^2\theta}{\sin\theta}=\frac{1}{\sin\theta}$ $\Rightarrow \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{1}{\sin\theta\cos\theta}$		
	(ii)	$\sin\theta\cos\theta=\frac{1}{4}\Rightarrow\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=4$ $\Rightarrow \tan\theta+\frac{1}{\tan\theta}=4$	M1 A1 **2**	Using **(i)** and tan
	(iii)	$\tan\theta+\frac{1}{\tan\theta}=4\Rightarrow\tan^2\theta+1=4\tan\theta$ $\Rightarrow t^2-4t+1=0$ $t=\frac{4\pm\sqrt{16-4}}{2}=2\pm\sqrt{3}\ \ (=3.732 \text{ and } 0.268)$ $\Rightarrow \theta=15^0 \text{ and } 75^0$ *Sp Case B1 for 15 and B1 for 75 with no supporting working*	M1 M1 A1 A1 **4**	Clear fractions to give 3 term quadratic Sub numbers into correct quadratic 3sf or more Rounds to these
8		$v=60\left(t^4-10t^3+25t^2\right)$ $\Rightarrow s=\int_0^5\left(60t^4-600t^3+1500t^2\right)\mathrm{d}t$ $=\left[12t^5-150t^4+500t^3\right]_0^5$ $=6250$ m If 60 is left out then 4/5 only.	M1 A2,1 DM1 A1 **5**	Integrate Terms 1 each error Sub $t=5$ Cao

9	**(i)**	Centre is $\left(\frac{1+15}{2},\frac{3+1}{2}\right)=(8,2)$ Nb Working with vectors to give diameter = [14,2] and so radius = [7,1] giving centre (15 – 7, 3 –1) is correct.	B1 B1 **2**	For 8 WWW For 2 WWW
	(ii)	$\lvert PC\rvert=\sqrt{(8-1)^2+(2-3)^2}=\sqrt{50}=5\sqrt{2}$	M1 A1 **2**	For $\sqrt{50}$
		Alt: Length of diameter = $\sqrt{(15-1)^2+(3-1)^2}=\sqrt{14^2+2^2}$ $=\sqrt{200}=10\sqrt{2}$ $\Rightarrow$ Radius = $5\sqrt{2}$		
	(iii)	$(x-8)^2+(y-2)^2=50$ $\Rightarrow x^2+y^2-16x-4y+64+4-50=0$ $\Rightarrow x^2+y^2-16x-4y+18=0$	M1 A1 **2**	Correct use of formula including 50 and using their midpoint.
10	**(i)**	Sub (0,4) Gives $k=\frac{1}{2}$	M1 A1 **2**	
	(ii)	Sub (0, 4) Gives $c=-\frac{1}{4}$	M1 A1 **2**	
	(iii)	When $x=3$ $y=-\frac{1}{4}(3-2)^2(3-4)=0.25$ for cubic Or when x= 3, $y>0$ for cubic John's model is better	B1 DB1 **2**	

Section B

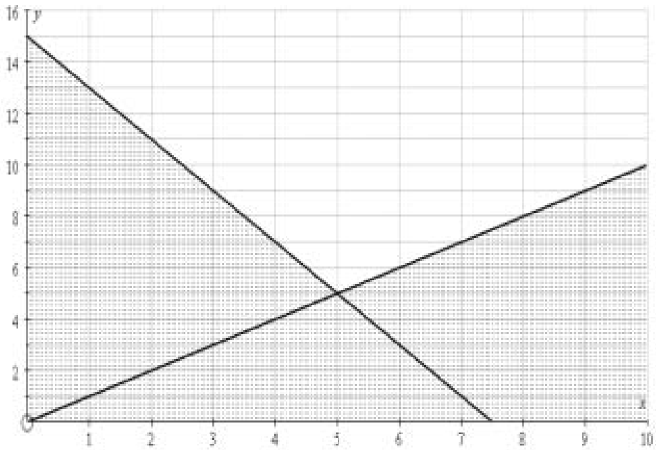
Allow 4 sf in this question

11	**(i)**	$\frac{AF}{\sin 70} = \frac{BF}{\sin 60} = \frac{100}{\sin 50}$ $\Rightarrow AF = \frac{100}{\sin 50} \times \sin 70 \ (= 122.7 \text{ m})$ $\Rightarrow BF = \frac{100}{\sin 50} \times \sin 60 = 113.1 \text{ m}$ oe	M1 A1 A1 M1 A1 **5**	Sin rule applied Sight of 50 and 70 Correct sine rule to find BF
		Alt: Cosine rule for BF: M1 $BF^2 = 100^2 + 122.7^2 - 2\times100\times122.7\times\cos 60$ $= 12785$ BF = 113.1 A1		
	(ii)	$FT = AF \times \tan 10$ $= 122.7 \tan 10 = 21.6 \text{ m}$ *Anything that rounds to 21.6*	M1 A1 **2**	
	(iii)	$CF = 122.7\sin 60$ $=106.3$ m Or: $=$ *their BF* $\times\sin 70$ $\Rightarrow \tan\theta = \frac{TheirFT}{TheirCF}$ $\Rightarrow \theta = 11.5^0$	M1 A1 M1 F1 A1 **5**	 Accept 106.2 or 106 Using tan correctly Substituting correctly Accept 11 or 12
		Alt: to find CF. Area of triangle $= \frac{1}{2}\times AF.AB\sin 60 = 5313$ M1 $\Rightarrow \frac{1}{2}\times CF\times 100 = 5313 \Rightarrow CF = 106.3$ A1		

12	**(i)**	$y = 0.3x^2 - 1.5x$ $\frac{dy}{dx} = 0.6x - 1.5$ When $x = 5$ $g_t = 1.5$ $\Rightarrow g_n = -\frac{2}{3}$ AB: $y = -\frac{2}{3}(x-5)$ $\Rightarrow 2x + 3y = 10$	B1 M1 A1 A1 **4**	Derivative Find g_t and use of $m_1 \times m_2 = -1$ For g_n Line in any simplified form
	(ii)	Solve simultaneously: $3y + 2x = 10$ $2y + 3x = 0$ $6y + 4x = 20$ $6y + 9x = 0$ $5x = -20$ $\Rightarrow x = -4, y = 6$ SC1: answer with no working	M1 F1 A1 **3**	Method to eliminate one variable x and y.
	(iii)	Area of triangle = $\frac{1}{2} \times 5 \times \textit{their } y = 15$ Area under curve = $\int_0^5 (0.3x^2 - 1.5x)\,dx$ $= \left[0.1x^3 - 0.75x^2\right]_0^5$ $= -6.25$ $\Rightarrow$ Area of card = $15 + 6.25 = 21.25$ *Other methods, follow scheme* *ieE1 Area of triangle* *M1 area as integral* *A1 Integrand* *A1 value for area* *A1 Final answer*	E1 M1 A1 A1 A1 **5**	Might appear anywhere in this part Ignore limits here Condone lack of –ve sign

13	(i)	Ali: $\frac{72}{t}$ Beth: $\frac{72}{t+2}$	M1 A1 A1 **3**	Accept Beth: $\frac{72}{t}-3$
	(ii)	$\frac{72}{t}-\frac{72}{t+2}=3$ $\Rightarrow 72(t+2)-72t=3t(t+2)$ $\Rightarrow 72t+144-72t=3t(t+2)$ $\Rightarrow 3t(t+2)=144$	M1 A1 M1 A1 A1 **5**	Subtraction of their terms = 3 Multiply out and simplify
		Alternative (based on alternative answer to (i) $\frac{72}{\frac{72}{t}-3}=t+2$ M1 A1 $\Rightarrow 72t=(72-3t)(t+2)$ $\Rightarrow 72t=72t-3t^2+144-6t$ M1 A1 $\Rightarrow 3t^2+6t=144 \Rightarrow 3t(t+2)=144$ A1		
	(iii)	$3t(t+2)=144$ $\Rightarrow 3t^2+6t-144=0$ $\Rightarrow t^2+2t-48=0$ $\Rightarrow (t+8)(t-6)=0$ $\Rightarrow t=6$ $\Rightarrow$ Ali takes 6 hours and Beth takes 8 hours. SC1 for answer with no working	M1 A1 A1 A1 **4**	For quadratic in simplified form. (See below) www

What is “simplified form”?
Either a quadratic with all three terms on left = 0 ready for the use of the formula
OR:
Divide through by 3 giving $t^2 + 2t = 48$ ready for solving by the completion of the square.

14	**(i)**	$200x + 100y \geq 1500$ oe	M1 A1 **2**	Deriving a linear inequality
	(ii)	$y \geq x$	B1 **1**	
	(iii)		B1 B1 E1 **3**	One line Other line Shading for both, ft their inequalities No Scales: B0, B0, E1 Condone scales not as instructed.
	(iv)	$C = 80x + 60y$ Correct point is (5, 5) Cost = £700 *In absence of OF, 80 ×5 + 60× 5 must be seen*	B1 B1 M1 A1 **4**	 Sub in OF
	(v)	Now minimum cost is at (7, 1) Giving £620 Nb (8, 0) gives £640	B1 B1 **2**	

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RECOGNISING ACHIEVEMENT

FREE-STANDING MATHEMATICS QUALIFICATION
ADVANCED LEVEL
Additional Mathematics — 6993

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 6993

Other materials required:
- Scientific or graphical calculator

Monday 13 June 2011
Morning

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES
These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES
This information is the same on the printed answer book and the question paper.

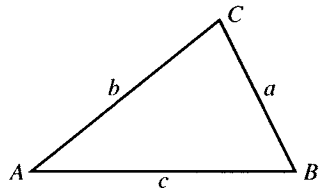
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- The printed answer book consists of **20** pages. The question paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR
- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Formulae Sheet: 6993 Additional Mathematics

In any triangle ABC

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$



Binomial expansion

When n is a positive integer

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Answer all questions on the Printed Answer Book provided.

Section A

1 Determine whether the point (5, 2) lies inside or outside the circle whose equation is $x^2 + y^2 = 30$. You must show your working. **[3]**

2 The equation of a curve is $y = x^3 - x^2 - 2x - 3$.

Find the equation of the tangent to this curve at the point (3, 9). **[5]**

3 In the triangle PQR, PQ = 8 cm, RQ = 9 cm and RP = 7 cm.

(i) Find the size of the largest angle. **[4]**

(ii) Calculate the area of the triangle. **[3]**

4 Solve the equation $5 \sin 2x = 2 \cos 2x$ in the interval $0° \leqslant x \leqslant 360°$.
Give your answers correct to 1 decimal place. **[5]**

5 The coordinates of the points A, B and C are (−2, 1), (5, 2) and (4, 9) respectively. M: 1,5

(a) Find the coordinates of the midpoint, M, of the line AC. **[1]**

(b) Show that BM is perpendicular to AC. **[3]**

(c) **(i)** Use the result of part **(b)** to state the mathematical name of the triangle ABC. **[1]**

(ii) Prove this by another method. **[2]**

6 Solve the inequality $x^2 - 12x + 35 \leqslant 0$. **[4]**

7 **(a)** Determine whether or not each of the following is a factor of the expression $x^3 - 7x + 6$.
You must show your working.

(i) $(x - 2)$ **[2]**

(ii) $(x + 1)$ **[1]**

(b) **(i)** Factorise the function $f(x) = x^3 - 7x + 6$. **[3]**

(ii) Solve the equation $f(x) = 0$. **[1]**

8 **(i)** On the axes given, indicate the region for which the following inequalities hold. You sho… shade the region which is **not** required.

$$5x + 3y \geqslant 30$$
$$3x + y \geqslant 12$$
$$y \geqslant 0$$
$$x \geqslant 0$$

[5]

(ii) Find the minimum value of $6x + y$ subject to these conditions. **[2]**

9 The gradient function of a curve is given by $\frac{dy}{dx} = 3x^2 - 2x + 4$.

Find the equation of the curve, given that it passes through the point (2, 2). **[4]**

10 You are given that $\sin\theta = \frac{2}{5}$ with $0° \leqslant \theta \leqslant 90°$.

Using the identity $\sin^2\theta + \cos^2\theta = 1$, find an exact value for $\cos\theta$. **[3]**

Section B

11 Eggs are delivered to a supermarket in boxes of 6.
For each egg, the probability that it is cracked is 0.05 independently of other eggs.

Find the probability that

(i) in one box there are no cracked eggs, **[2]**

(ii) in one box there is exactly 1 cracked egg. **[4]**

The manager checks the eggs as follows.

- He takes a box at random from the delivery.
- He accepts the whole delivery if this box contains no cracked eggs.
- He rejects the whole delivery if the box contains 2 or more cracked eggs.
- If the box contains 1 cracked egg then he chooses another box at random.
- He accepts the delivery only if this second box contains no cracked eggs.

(iii) Find the probability that the delivery is rejected. **[6]**

12 Two cars, A and B, move from rest away from a point O on a straight road starting at the same time.

(a) Car A moves with constant acceleration of 2 m s^{-2}.

Express the displacement of car A after time t seconds as a function of t. **[2]**

(b) Car B moves with acceleration given by $a = \frac{1}{2}t + 1$.

Express the displacement of car B after time t seconds as a function of t. **[4]**

(c) **(i)** Find the time at which the cars are the same distance from O. **[2]**

(ii) Find the distance they have travelled at that time. **[2]**

(d) Draw a sketch graph of the velocity of each car on the axes given. **[2]**

13 A pyramid has a square base, ABCD, with vertex E. E is directly above the centre of the base, O, as shown in Fig. 13.
The lengths of the sides of the base are each $2x$ metres and the height is h metres.
The lengths of the sloping edges, AE, BE, CE and DE, are each 5 metres.

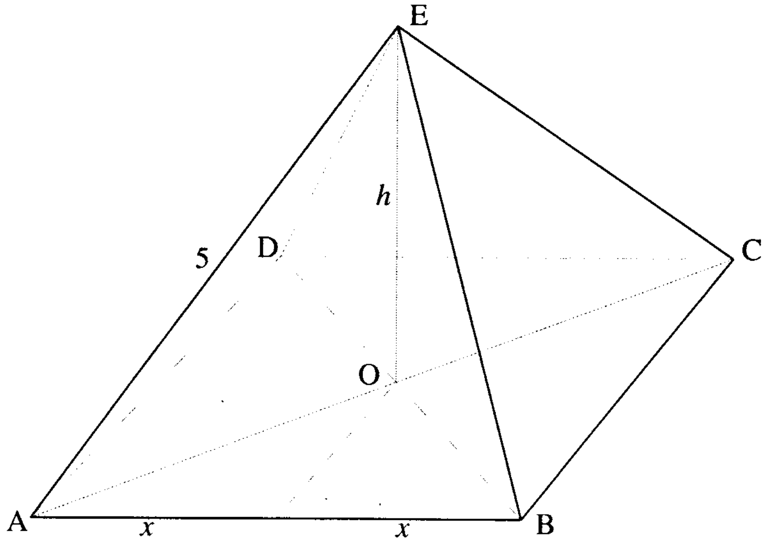


Fig. 13

(i) Show that $2x^2 = 25 - h^2$. **[2]**

(ii) Show that the volume of the pyramid, V m^3, is given by $V = \dfrac{50h - 2h^3}{3}$. **[2]**

(iii) As h varies, find the value of h for which V has a stationary value. **[4]**

(iv) Prove that this stationary value is a maximum. **[2]**

(v) Calculate the angle between the edge AE and the base when h takes this value. **[2]**

[Volume of a pyramid $= \frac{1}{3} \times$ base area $\times$ height.]

14 The cross-section of a speed hump is modelled by the region enclosed by the x-axis and the curve

$$y = \frac{1-(x-1)^4}{5}.$$

The graph is shown in Fig. 14.
Units are metres.

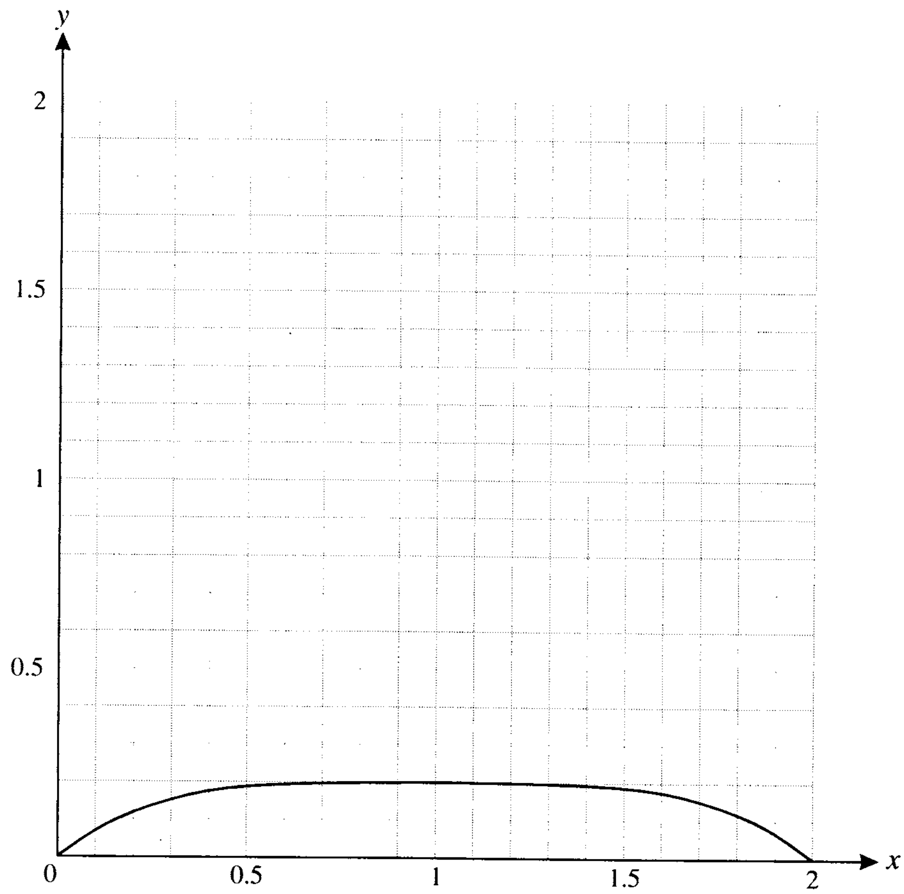


Fig. 14

(a) **(i)** Write down the maximum value of $1-(x-1)^4$. **[1]**

(ii) Hence write down the maximum height of the speed hump. **[1]**

(b) Show that $y = \frac{1}{5}(4x - 6x^2 + 4x^3 - x^4)$. **[3]**

(c) Find the area of the cross-section of the speed hump. **[7]**

RECOGNISING ACHIEVEMENT

FSMQ

Additional FSMQ

Free Standing Mathematics Qualification

6993: Additional Mathematics

Mark Scheme for June 2011

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Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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1. **Subject-specific Marking Instructions**

1. **M** (method) marks are not lost for purely numerical errors.
A (accuracy) marks depend on preceding **M** (method) marks. Therefore **M0 A1** cannot be awarded.
B (independent) marks are independent of **M** (method) marks and are awarded for a correct final answer or a correct intermediate stage.

2. Subject to 1, two situations may be indicated on the mark scheme conditioning the award of **A** marks or **B** marks:
   i. Correct answer correctly obtained (no symbol)
   ii. Follows correctly from a previous answer whether correct or not (**FT** on mark scheme and on the annotations tool).

3. Always mark the greatest number of significant figures seen, even if this is then rounded or truncated in the answer.

4. Where there is clear evidence of a misread, a penalty of 1 mark is generally appropriate. This may be achieved by awarding **M** marks but not an **A** mark, or awarding one mark less than the maximum.

5. For methods not provided for in the mark scheme give as far as possible equivalent marks for equivalent work. If in doubt, consult your team leader.

6. Where a follow through (**FT**) mark is indicated on the mark scheme for a particular part question, you must ensure that you refer back to the answer of the previous part question if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

2. **Abbreviations**

The following abbreviations are commonly found in Mathematics mark schemes.

- Where you see **oe** in the mark scheme it means **or equivalent.**
- Where you see **cao** in the mark scheme it means **correct answer only.**
- Where you see **soi** in the mark scheme it means **seen or implied.**
- Where you see **www** in the mark scheme it means **without wrong working.**
- Where you see **rot** in the mark scheme it means **rounded or truncated.**

- Where you see **seen** in the mark scheme it means that you should award the mark if that number/expression is seen anywhere in the answer space, even if it is not in the method leading to the final answer.

- Where you see **figs 237**, for example, this means any answer with only these digits. You should ignore leading or trailing zeros and any decimal point e.g. 237000, 2·37, 2·370, 0·00237 would be acceptable but 23070 or 2374 would not.

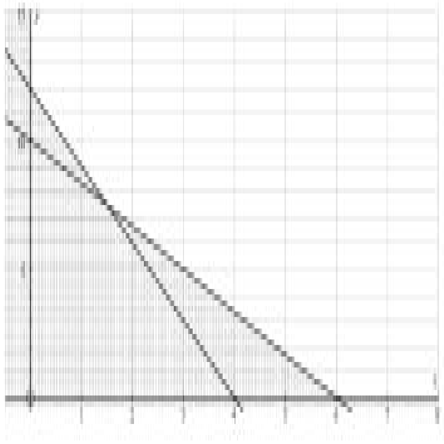
.

Section A

Question			Answer	Marks	Part Marks and Guidance	
1			For (5, 2) use $x^2 + y^2$ $= 29$ so inside	M1 A1 A1 3	Substitute or use Pythagoras **soi** or √29 Conclusion (dependent on M1A1 awarded)	As usual only award A marks if the M mark has been awarded. **Alternative method:** Sub of $x = 5$ or $y = 2$ in $x^2 + y^2 = 30$ to find y or x **M1** $y = \sqrt{5}$ or $x = \sqrt{26}$ **A1**
2			$\frac{dy}{dx} = 3x^2 - 2x - 2$ At $x = 3$ gradient $= 27 - 6 - 2 = 19$ $\Rightarrow y - 9 =$ "their" $19(x - 3)$ $\Rightarrow y = 19x - 48$ oe	M1 A1 A1 M1 A1 5	Differentiate All three terms correct 19 **isw** (**dep** on first **M1**) Find line using correct point and *their* 19	At least one power decreased by 1. "*their*" 19 means: the value of the derivative Only 3 terms
3	i		eg $\cos P = \frac{8^2 + 7^2 - 9^2}{2.8.7}$ oe $\Rightarrow P = 73.4°$	M1 A1 M1 A1 4	Cosine formula correctly used to find any angle Anything that rounds to 73.4°, 48.2° or 58.4° For identifying correct angle	 Anything that rounds to 73.4°

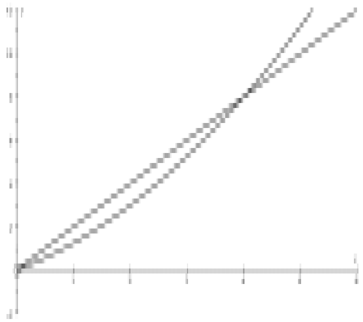
Question			Answer	Marks	Part Marks and Guidance	
3	ii		Area $=\frac{1}{2}\times 7\times 8\times \sin(\textit{their angle } P)$ $=26.8$	**M1** **A1** **A1** **3**	Use of formula Correct substitution from *their* **(i)**	Can be at any vertex Anything that rounds to 26.8 Accept complete alternative methods
4			$5\sin 2x = 2\cos 2x$ $\Rightarrow \tan 2x = 0.4$ $\Rightarrow 2x = 21.8,\ 201.8$ $\Rightarrow x = 10.9,\ 100.9$ Also $x = 190.9,\ 280.9$	**M1** **A1** **A1** **A1** **A1** **5**	Use of tan 0.4 10.9 Any 2^{nd} value 3^{rd} and 4^{th} values (*ignore extra solutions)*	allow tan*x* = 0.4 for **M1A1** **Alternative method** Use of Pythagoras to get $\sin 2x=\frac{2}{\sqrt{29}}$ or $\cos 2x=\frac{5}{\sqrt{29}}$ **M1A1** *and the last three marks are still available, ignore extra solutions*
5	a		*M is* $\left(\frac{-2+4}{2},\frac{1+9}{2}\right)$ *which is* $(1,5)$	**B1** **1**		
5	b		Gradient of AC is $\frac{9-1}{4+2}=\frac{4}{3}$ Gradient of BM is $\frac{2-5}{5-1}=-\frac{3}{4}$ $\frac{4}{3}\times -\frac{3}{4}=-1$ oe	**B1** **B1** **B1** **3**	One gradient Second gradient *Their* $m_1\times m_2=-1$	 Eg One is the negative reciprocal of the other
5	c	i	Isosceles	**B1** **1**	Allow right-angled isosceles	Accept wrong spelling Do not accept right-angled triangle

Question			Answer	Marks	Part Marks and Guidance	
5	c	ii	$AB^2 = 7^2 + 1^2 = 50$ $BC^2 = 7^2 + 1^2 = 50$ $\Rightarrow$ two sides equal in length	M1 A1 2	Using Pythagoras on AB and BC Or fully labelled diagram with correct sides shown	Attempt by vectors **AB** and **BC M1** **Alternative** If answer to **(c)(i)** was right-angled, then accept proof that it is (requires all three lengths.) **Alternative:** If **(c)(i)** was equilateral or scalene then **M1** (only) for attempt at all three sides. NB If nothing is written in **(i)** then no credit in this part.
6			$(x \pm 5)(x \pm 7)$ Boundaries $x = 5, x = 7$ $\Rightarrow 5 \le x \le 7$	M1 A1 B2 4	Or use of correct formula (allow one error in substitution) or correct shaped graph seen **soi** Accept $x \ge 5$, $x \le 7$ for **B1, B1**	Condone < or >
7	a	i	Attempt to find f(2) by substitution of 2 = 0, So Yes	M1 A1 2	Remainder theorem or attempt to divide (justification is sight of $x^3 - 2x^2$) Or: attempt to factorise, justification is sight of $(x^2 \ldots 3)$ Correct working only	
7	a	ii	f(−1) = −1 + 7 + 6 = 12 so no.	B1 1	Sight of 12 or correct evidence, conclusion required	
7	b	i	$f(x) = (x - 2)(x^2 + 2x - 3)$ $= (x - 2)(x + 3)(x - 1)$	M1 A1 A1 3	Attempt to factorise or use long division (justifications as in **(a)(i)**) Sight of correct quadratic **soi** Answer	**Alternative:** Use of Remainder theorem **M1** Sight of 2nd factor **A1** All correct **A1**
7	b	ii	$x = 1, 2, -3$	B1 1	**FT** their brackets	Must be three roots

Question			Answer	Marks	Part Marks and Guidance	
8	i			B1 B1 B1 B1 B1 5	for one line for correct shading for other line for correct shading for correct shading to give $x \geq 0$, $y \geq 0$ NB If intercepts are within 1 small square of the correct points then give the marks for the lines	*If there is work here that is not crossed out, then mark it and ignore anything on Page 18.* Helpful hint: Lines go through (0, 12) and (4,0) (0, 10) and (6, 0) Intersection at (1.5, 7.5) If **B0** for a line allow **B1** for shading if negative gradient and lines intersect
8	ii		$6x + y$ is minimum at (0, 12) *(can be implied by correct answer)* So is 12	B1 B1 2		
9			$\frac{dy}{dx} = 3x^2 - 2x + 4 \Rightarrow (y) = x^3 - x^2 + 4x + c$ (2,2) satisfies $\Rightarrow 2 = 8 - 4 + 8 + c$ $\Rightarrow c = -10$ $\Rightarrow y = x^3 - x^2 + 4x - 10$	M1 A1 M1 A1 4	Integrate Ignore c (**dep** on 1st **M1** mark) Substitute **cao**	At least one term with power increased by 1. (NB do not accept multiplying throughout by x) (ie must be $y = \ldots$.)
10			$\sin\theta = \frac{2}{5} \Rightarrow \sin^2\theta = \frac{4}{25} \Rightarrow 0.16 + \cos^2\theta = 1$ $\cos^2\theta = \frac{21}{25}$ $\Rightarrow \cos\theta = \frac{1}{5}\sqrt{21}$ oe	M1 A1 A1 3	Use of Pythagoras $\cos^2\theta$ eg $\sqrt{0.84}$ or $\sqrt{\frac{21}{25}}$ **isw**	Sight of a triangle with sides 2, 5, $\sqrt{21}$ acceptable for **M1** Then **A2** for $\cos\theta$ NB **M0** if calculator used to find θ in order to find $\cos\theta$

Section B

Question			Answer	Marks	Part Marks and Guidance	
11	**i**		P(0) = $(0.95)^6$ = 0.735(09189…)	**M1** **A1** **2**	Correct *p* plus correct power	 Not 2sf
11	**ii**		P(1) = 6 × $(0.95)^5$ × $(0.05)^1$ = 0.232(134281…)	**M1** **B1** **B1** **A1** **4**	Correct *p* and *q* and powers add to 6 Coefficient **soi** Correct powers for correct *p* and *q* **soi**	Coefficient may be missing
11	**iii**		P(1st box contains 2 or more eggs) = 1 − (their**(i)** + their **(ii)**) = 1 − (0.7351 + 0.2321) =1 − 0.9672 = 0.0328 P(2nd box has any cracked eggs) = 1 − *their* **(i)** = 0.2649 P(consignment is rejected) = 0.0328 + 0.2649 × their **(ii)** = 0.0328 + 0.0615 = 0.0943	**M1** **A1** **M1** **A1** **M1** **A1** **6**	 Accept anything rounding to 0.033 Accept anything rounding to 0.265 In either method, accept answers which lie between 0.094 and 0.095	Alternative P(accepted) **M1** Ans**(ii)** × Ans**(i)** **A1** 0.1706 **soi** (Accept 0.171) **M1(dep)** Add to this Ans**(i)** **A1** 0.9057 (Accept 0.906) **M1** P(consignment is rejected) = 1 − 0.9057 **A1** = 0.09428

Question			Answer	Marks	Part Marks and Guidance	
12	a		$s = ut + \frac{1}{2}at^2$ with $u = 0$ and $a = 2$ $\Rightarrow s = t^2$	M1 A1 2	Constant acceleration formulae or integrate twice – ignore *c*	
12	b		$(v) = \frac{t^2}{4} + t$ $s = \frac{t^3}{12} + \frac{t^2}{2}$ Ignore *c*	M1 A1 M1 A1 4	Integrate Integrate	
12	c	i	$\frac{t^3}{12} + \frac{t^2}{2} = t^2$ $\Rightarrow \frac{t}{12} + \frac{1}{2} = 1$ $\Rightarrow t = 6$	M1 A1 2	Equate their functions	
12	c	ii	$s = 6^2$ or $s = \frac{6^3}{12} + \frac{6^2}{2}$ Displacement = 36 (m)	M1 A1 2	Substitute *their non-zero* **(c)(i)** in *their* **(a)** or **(b) soi**	
12	d			B1 B1 2	One clearly straight line through origin with positive gradient Other clearly a curve through the origin of correct shape with first part below the line as per diagram	Ignore labels

Question			Answer	Marks	Part Marks and Guidance	
13	i		$AO^2 = x^2 + x^2 = 2x^2$ or $AC^2 = (2x)^2 + (2x)^2 = 8x^2$ $h^2 + AO^2 = AE^2 \Rightarrow h^2 + 2x^2 = 25$ $\Rightarrow 2x^2 = 25 - h^2$	M1 A1 2	Correct application of Pythagoras on the base Algebra must be convincing	NB Answer is given
13	ii		$V = \frac{1}{3} \times \text{base area} \times \text{height} = \frac{1}{3} \times 4x^2h$ $= \frac{50h - 2h^3}{3}$	M1 A1 2	Formula seen including $4x^2$	Care: the answer is given
13	iii		$\frac{dV}{dh} = \frac{50 - 6h^2}{3}$ $= 0$ when $50 - 6h^2 = 0$ $\Rightarrow h^2 = \frac{25}{3}$ $\Rightarrow h = \sqrt{\frac{25}{3}} = \frac{5}{\sqrt{3}} = \frac{5}{3}\sqrt{3} = 2.89$	M1 A1 M1 A1 4	Differentiation **cao** **dep** Set (numerator) = 0 Any of these answers is acceptable	**SC3** h = 2.89 with either $\frac{dV}{dh}$ or $\frac{1}{3}$ missing Numerical value must be 2.89
13	iv		$\frac{d^2V}{dh^2} = -4h$ < 0 so maximum	M1 A1 2	Or alternatives: Complete method to investigate value of derivative Or: complete method to investigate the value of V either side **and** at the turning point	Accept $-12h$
13	v		At this point sinEAO $= \frac{h}{5} = \frac{1}{3}\sqrt{3}$ $\Rightarrow$ Angle EAO = 35.3°	M1 A1 2	Use of a correct ratio with *their h* (and/or *x*) Accept 35.2 which comes from h = 2.88	

Question			Answer	Marks	Part Marks and Guidance	
14	a	i	Max value = 1	B1 1		Not from any use of 0.2 from graph
14	a	ii	Height = 0.2 (m) or 20 cm	B1 1		
14	b		$x^4 - 4x^3 + 6x^2 - 4x + 1$ $\Rightarrow y = \frac{1}{5}\left(4x - 6x^2 + 4x^3 - x^4\right)$	B2 B1 3	-1 each error **Dep on B2** convincing algebra (means sight of an extra correct step **www**)	An error is signs, powers, coefficients, failure to include the 1 at end
14	c		Area = $\int_0^2 \frac{1}{5}\left(4x - 6x^2 + 4x^3 - x^4\right).dx$ $= \frac{1}{5}\left[2x^2 - 2x^3 + x^4 - \frac{x^5}{5}\right]_0^2$ $= \frac{1}{5}\left(8 - 16 + 16 - \frac{32}{5}\right) = \frac{8}{25}$ $= 0.32$ Area of cross section = 0.32m^2 = 3200cm^2	M1 A3 M1 A1 A1 7	Integrate (ignore *c*) **A2** if one error, **A1** if two errors (**Dep** on 1st **M1**) Deal with limits correctly (Putting x = 0 does not need to be seen) Units	**Alternative method:** Integrate original function is OK, but in dealing with limits x = 0 must then be seen. Omission of $\frac{1}{5}$ is one error. Multiply by $\frac{1}{5}x$ or $\frac{1}{5x}$, ie integrating $\frac{1}{5}$ gives **A0**

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RECOGNISING ACHIEVEMENT

Wednesday 30 May 2012 – Afternoon

FSMQ ADVANCED LEVEL

6993 Additional Mathematics

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 6993

Other materials required:
- Scientific or graphical calculator

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

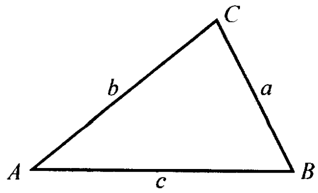
INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Formulae Sheet: 6993 Additional Mathematics

In any triangle *ABC*

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$



Binomial expansion

When n is a positive integer

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Section A

1 **(i)** Find the range of values of x satisfying $x^2 - 4x + 3 \leqslant 0$. **[3]**

(ii) Show this range on the number line provided. **[1]**

2 A die has 6 faces numbered one to six. The die is biased so that when it is thrown the probability of obtaining a six is $\frac{1}{5}$.

The die is thrown 5 times.

Find the probability of obtaining

(i) at least 1 six, **[2]**

(ii) exactly 3 sixes. **[4]**

3 The function $f(x) = x^3 + ax + 6$ is such that when $f(x)$ is divided by $(x - 3)$ the remainder is 12.

(i) Show that the value of a is -7. **[2]**

(ii) Factorise $f(x)$. **[3]**

4 A car moves from rest with constant acceleration on a straight road. When the car passes a point A it is travelling at $10\,\mathrm{m\,s^{-1}}$ and when it passes a point B further along the road it is travelling at $16\,\mathrm{m\,s^{-1}}$.

The car takes 10 seconds to travel from A to B.

Find

- the distance AB,
- the constant acceleration. **[4]**

5 **(i)** Show that the equation $3\cos^2\theta = \sin\theta + 1$ can be written as $3\sin^2\theta + \sin\theta - 2 = 0$.

(ii) Solve this equation to find values of θ in the range $0° < \theta < 360°$ that satisfy

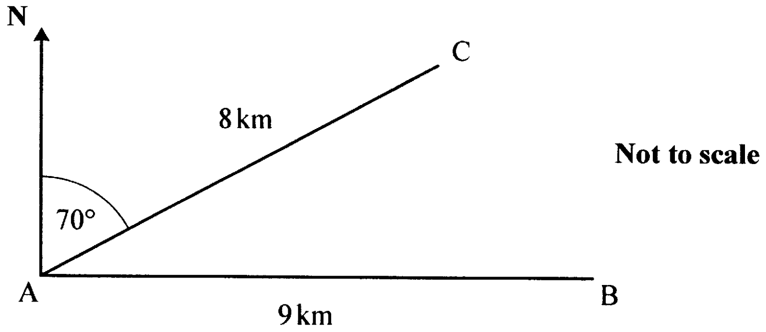
$$3\cos^2\theta = \sin\theta + 1 .$$ **[4]**

6 The equation of a curve is $y = 2x^3 - 9x^2 + 12x$.

(i) Show that the curve has a stationary point where $x = 2$. **[4]**

(ii) Determine whether the stationary value where $x = 2$ is a maximum or minimum. **[2]**

7 A yachtsman wishes to sail from a port, A, to another port, B, which is 9 km due East of A.
Because of the wind he is unable to sail directly East and sails 8 km on a bearing of 070° to point C.



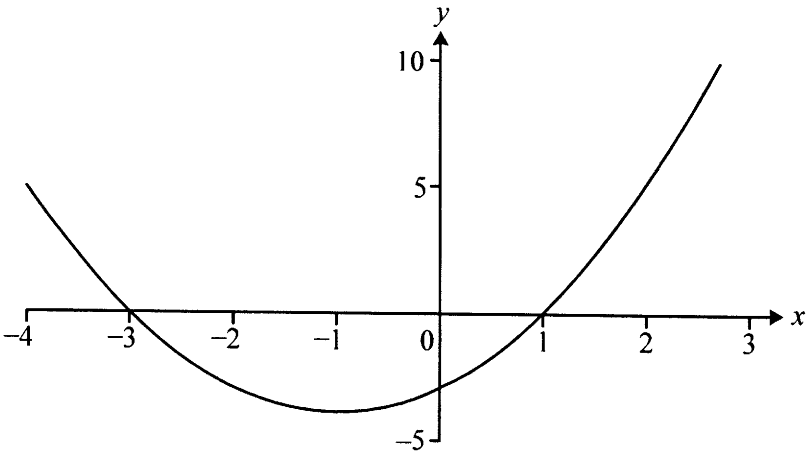
Calculate

(i) the distance he is now from port B, **[3]**

(ii) the angle ABC and hence the bearing on which he must sail to reach port B from point C, correct to the nearest degree. **[4]**

8 **(i)** Show that $\int_0^2 (x^2 + 2x - 3)\,dx = \frac{2}{3}$. **[3]**

The diagram shows part of the curve $y = x^2 + 2x - 3$.



(ii) Marc claims that the total area between the curve, the x-axis and the lines $x = 0$ and $x = 2$ is $\frac{2}{3}$.

Explain why he is wrong. **[1]**

(iii) Calculate the total area between the curve, the x-axis and the lines $x = 0$ and $x = 2$. **[3]**

9 The height above the ground of a seat on a fairground big wheel is h metres. At time t **minutes** after the wheel starts, h is given by

$$h = 7 - 5\cos(480t)°.$$

(i) Write down the initial height above the ground of the seat (when $t = 0$). **[1]**

(ii) Find the greatest height reached by the seat. **[2]**

(iii) Calculate the time of the first occasion when the seat is 9 metres above the ground. Give your answer correct to the nearest second. **[4]**

Section B

10 A (1, 10), B (8, 9) and C (7, 2) are three points.

(i) Find the coordinates of the midpoint, M, of AC. [1]

(ii) Find the equation of the circle with AC as diameter. [4]

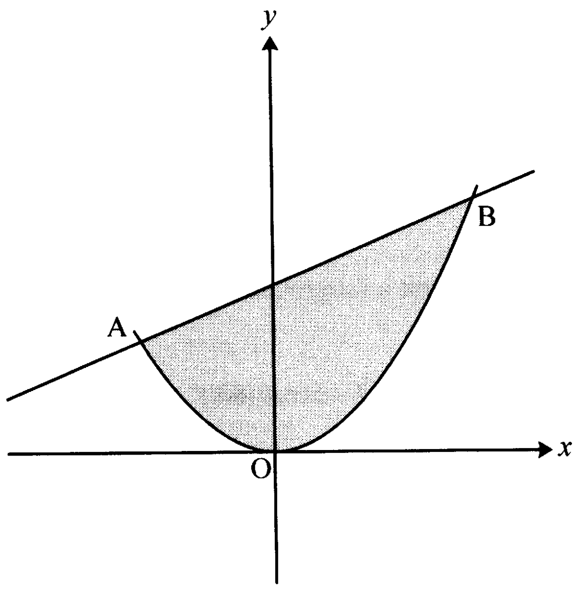
(iii) Show that B lies on this circle. [1]

(iv) Prove that AM and BM are perpendicular. [3]

(v) BD is a diameter of this circle. Find the coordinates of D. [3]

11 The shaded region in the diagram shows a wooden shape.

The curve has equation $y = \frac{1}{2}x^2$ and the coordinates of A are (−2, 2).



The line AB is the normal to the curve at the point A.

(i) Find the equation of the line AB. [5]

(ii) Find the coordinates of the point B where the line AB meets the curve again. [3]

(iii) Find the shaded area. [4]

12 The Highway Code gives a table of shortest stopping distances (d feet) for a vehicle travelling at v miles per hour.

The formula used for this table is given by

$$d = av^2 + bv\,.$$

Two entries in the table are given below.

v **mph**	d **feet**
30	75
60	240

(i) By forming and solving a pair of simultaneous equations in a and b, show that the formula is

$$d = \frac{v^2}{20} + v.$$ **[5]**

(ii) Find the difference between the stopping distances for a car travelling at 65 mph and a car travelling at 70 mph. **[3]**

(iii) Many drivers maintain a distance of 50 feet or less when driving on a motorway.

Use the formula in part **(i)** to find the speed at which the shortest stopping distance is 50 feet. **[4]**

Question 13 is printed overleaf

13 **(i)** Find the coefficients a, b and c in the expansion

$$(2 + h)^3 = 8 + ah + bh^2 + ch^3 .$$ **[3]**

(ii) The graph of the equation $y = x^3$ passes through the points P and Q which have x-coordinates 2 and $2 + h$ respectively.

Show that the gradient of the chord PQ is $\dfrac{(2+h)^3 - 8}{h}$. **[3]**

(iii) Express $\dfrac{(2+h)^3 - 8}{h}$ as a quadratic function of h. **[2]**

(iv) As the value of h decreases, the point Q gets closer and closer to the point P on the curve. As h gets closer to 0 the chord PQ gets closer to being the tangent to the curve at P.

Deduce the value of the gradient of the tangent at P. **[1]**

(v) Kareen uses the same method to deduce the value of the gradient of the tangent at the point (2, 16) on the curve $y = x^4$.

The first three lines of her working are given below and in the answer booklet.

Take P to be the point (2, 16)

Take Q to be the point $(2 + h, (2 + h)^4)$

The gradient of the chord PQ is given by $\dfrac{(2+h)^4 - 16}{h} =$

Complete Kareen's working. **[3]**

RECOGNISING ACHIEVEMENT

FSMQ

Additional FSMQ

Free Standing Mathematics Qualification

6993: Additional Mathematics

Mark Scheme for June 2012

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

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Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

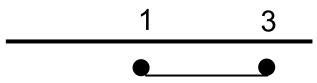
Viewing tips for this paper

In general, set your screen to 'fit width.'

You may find it helpful to set to 'fit height' for the some questions:

[if you set a view, it stays for subsequent scripts]. If the writing is too small, you may wish to zoom in.

Section A

Question			Answer	Marks	Guidance	
1	(i)		$(x \pm 1)(x \pm 3)$ (≤ 0) $\Rightarrow 1 \leq x \leq 3$ **www**	**M1** **A1** **A1** **[3]**	Attempt to factorise **oe** Correct (or sight of 1 & 3) Answer	**SC** Test integers and select 1 and 3 **B**1 Accept $x \leq 3$ **and** $1 \leq x$ Or : from 1 to 3 **inclusive** (must imply **inclusion** of end points).
			Alternative: Draw curve for parabola the right way up **M1** Correct points on x-axis **A1** answer **A1**			
	(ii)			**B1** **[1]**	Correct answer Or: Follow through their ans to **(i)**.	Filled in circles must be evident. **SC B1** if correct but M0 in **(i)**. Accept alternative conventions. Answer must be a range (ie just a set of points is 0).

Question			Answer	Marks	Guidance	
2	(i)		$=1-\left(\frac{4}{5}\right)^5$	M1	$1-p^5$	p does not have to be 0.8 for this mark but the power must be 5. (ie p could be 0.2)
			$=0.672(32)=\frac{2101}{3125}$	A1 [2]		
			Alternative: P(1) + ...+ P(5) 5 terms added, each term with powers correct **M1** Answer **A1**		Condone missing coeffs for **M1**	Terms are: 0.4096, 0.2048, 0.0512, 0.0064, 0.00032.
	(ii)		$10p^3q^2=10\times0.2^3\times0.8^2$	M1	Must include powers of p and q and $\binom{5}{3}$ or ${}^5C_3$ (which need not be evaluated	Can be obtained by listing.
				B1 A1	Powers Coefficient **soi**	
			$=0.0512=\frac{32}{625}$ **www**	A1 [4]	Accept 0.051 but not 0.05	

Question			Answer	Marks	Guidance	
3	(i)		f(3) = 12 ⇒ 27+3a + 6 = 12 ⇒3a = –21 ⇒ a = –7	M1 A1 [2]		
			Alternative: Substitute a = –7 **M1** and show that R = 12 **A1**			If this method is used then if long division is used then $x^3 - 3x^2$ must be seen. NB Answer given so long division must be totally correct for A1
	(ii)		f(1) = 0 or (x – 1) seen ⇒ f(x) = (x – 1)(x – 2)(x + 3)	M1 A1 A1 [3]	Divide, try factor theorem for at least one value, or obtain a 3-term quadratic factor by inspection. Using or getting a correct factor or root Answer	Divide means you need to see the x^2 in the quotient and x^3 and x^2 terms correct in the initial dividing line.

Question			Answer	Marks	Guidance	
4			$s=\left(\frac{u+v}{2}\right)t \Rightarrow s=13\times10=130$ **www**	**M1** **A1**	In any order using any valid formulae. Ignore units	eg $s=\frac{(10+16)}{2}\times10$
			$v=u+at \Rightarrow a=\frac{16-10}{10}=0.6$ **www**	**M1** **A1** **[4]**		eg $16=10+10a$ Alternative order: $a=\frac{16-10}{10}=0.6$ $\Rightarrow s=10\times10+\frac{1}{2}\times0.6\times10^2=130$ **MR** u = 0 and v = 10 gives s = 50, a = 1 Or u= 0 and v= 16 gives s = 80 and a = 1.6 **M1 A0 M1 A0**

Question			Answer	Marks	Guidance	
5	(i)		$3-3\sin^2\theta = \sin\theta + 1$ $\Rightarrow 3\sin^2\theta + \sin\theta - 2 = 0$ **www**	**M1** **A1** **[2]**	Sight of and use of $\cos^2\theta = 1 - \sin^2\theta$ Must see = 0	NB answer given
	(ii)		$(3\sin\theta - 2)(\sin\theta + 1) = 0$ $\Rightarrow \sin\theta = -1$ or $\sin\theta = \frac{2}{3}$ $\Rightarrow \theta = 270°, \ 41.8°, \ 138.2°$	**M1** **A1** **A2** **[4]**	Solve to obtain $\sin\theta = \pm 1$ or $\sin\theta = \pm\frac{2}{3}$ Sight of both values All 3 with no extras in range Ignore –90° **A1** for one or two values Or: all 3 values correct but extra values in range. Anything that rounds to 41.8° and 138° Allow 138° but not 42°	**SC2** $\sin\theta = 1, -\frac{2}{3}$ $\Rightarrow \theta = 90°, 318.2°, 221.8°$ (only) (Allow 318° and 222°)

Question			Answer	Marks	Guidance	
6	(i)		$\frac{dy}{dx} = 6x^2 - 18x + 12$	M1 A1	Differentiation All three terms	At least 2 terms with powers reduced by 1 (NB: beware division by *x*).
			When $x = 2, \frac{dy}{dx} = 24 - 36 + 12 = 0$	M1 dep A1 [4]	Sub *x* = 2 into or factorise their derived function. Get 0 or set = 0 and get 2.	**Do not condone division by 6 before substituting for *x*.** **NB answer given. Numerical values must be seen** Second M1 dep on first M1
	(ii)		$\frac{d^2y}{dx^2} = 12x - 18$	M1	Diffn their derived function correctly.	Using the function 2*x* – 3 can earn M1 A0.
			When $x = 2, \frac{d^2y}{dx^2} > 0$ giving a minimum	A1 [2]	**BOD** no arithmetic computations seen.	
			Alternative: Sign of gradient either side of *x* = 2 **M1** **Or:** Values of *y* either side of *x* = 2 **and** the value of *y* at stationary point. **M1** Correct answer (provided l.h. *x* > 1) **A1**		**BOD** no arithmetic computations seen. For **A1** LH *x* greater than 1	**Allow** sketch of function indicating left stationary value is maximum and right one is minimum**.**

Question		Answer	Mark	Guidance	
7	(i)	$(CB^2 =)\ 8^2 + 9^2 - 2\times 8\times 9\times \cos 20$ $= 9.684$ $\Rightarrow CB = 3.11$	M1 A1 A1 [3]	8, 9 must be used, any angle **soi** Anything that rounds to 3.11	Ignore units
	(ii)	$\frac{\sin ABC}{8} = \frac{\sin \textit{their}\ 20}{\textit{their}\ 3.11}$ $\Rightarrow \sin ABC = 0.879$ $\Rightarrow ABC = 61.55°$ $\Rightarrow$ Bearing = 152°	M1 A1ft A1 A1ft [4]	Correct application of sine rule Must be same angle as used in **(i)** and *their* CB Anything that rounds to 62° **www** Anything that rounds to 152°	90 + their ABC
		Alternative methods: Cosine Rule: $\cos ABC = \frac{9^2 + \textit{their}\ CB^2 - 8^2}{2\times 9\times \textit{their}\ CB}$ $= 0.4767$ **M1 A1ft** Then angle and bearing **A1 A1ft** **OR**: Perpendicular from C and use of sin twice **M1** $h = 8\sin \textit{their} 20 = 2.736$ **A1ft** $\text{Sin}ABC = \frac{2.736}{\textit{their}CB}$ Then angle and bearing **M1 A1ft** **Or:** Find other angle by sine rule **M1 A1** Angle ACB = 98.45 giving ABC = 61.55 **A1** Bearing = 180 – (98.45 – 70) = 152 **A1ft**		Correct application of cos rule Must be same angle as used in **(i)** and *their* CB	NB Question asks for ABC so if not found 3/4 Angle = 81.55 can earn M1 A0 A0 (for ABC) A1ft only

Question			Answer	Marks	Guidance	
8	(i)		$\int_0^2 (x^2+2x-3)\,dx = \left[\frac{x^3}{3}+x^2-3x\right]_0^2$ $=\left(\frac{8}{3}+4-6\right)-(0)$ **oe** $=\frac{2}{3}$ **www**	M1 A1 A1 [3]	Integrate All three terms Completion to $\frac{2}{3}$.	Test for integration is "are there at least two terms with the power increased by 1?" Care that the process is not just multiplying each term by *x*. Working must be seen as the answer is given. Ignore absence of "– 0".
	(ii)		Because the curve crosses the *x*-axis in the range	B1 [1]	Because one bit is +ve and the other is –ve.	Any reference to *x*= -3 will be 0. If there is an additional statement give 0.
	(iii)		$\left[\frac{x^3}{3}+x^2-3x\right]_0^1$ or $\left[\frac{x^3}{3}+x^2-3x\right]_1^2$ $=\pm 1\frac{2}{3}$ or $2\frac{1}{3}$ $\Rightarrow$ Total area $= 1\frac{2}{3}+2\frac{1}{3}=4$	M1 A1 A1 [3]	Calculation of their integral between 0 & 1 or 1& 2 One of the areas	

Question			Answer	Marks	Guidance	
9	(i)		$h = 7 - 5 \times \cos 0 = 2$	**B1** **[1]**		
	(ii)		$h = 7 - (-5) = 12$	**M1** **A1** **[2]**	Set $\cos\theta = -1$	
	(iii)		$9 = 7 - 5\cos(480t)$ $\Rightarrow \cos(480t) = -0.4$ **oe** $\Rightarrow 480t = 113.578$ $\Rightarrow t = 0.2366$ $\Rightarrow$ time = 0.2366 mins = 14 sec	**M1** **A1** **A1** **A1** **[4]**	Substitute $h = 9$ **soi**	Allow 114 leading to $t = 0.2375$

Section B

Question			Answer	Marks	Guidance	
10	(i)		$(4,6)$	B1 [1]		
	(ii)		Distance MC: $\sqrt{(4-7)^2+(6-2)^2}$ $=5$ Equation of circle: $(x-4)^2+(y-6)^2=5^2\ (=25)$	M1 A1 M1 A1 [4]	Attempt to find radius or diameter by pythagoras. **soi** Must include their M and their r^2 Can be expanded form.	Answer given with no working then bod **B2** (ie r = 5, r^2 = 25, d = 10, d^2 = 100)
			Alternative: Equation of circle on AC as diameter: $(x-1)(x-7)+(y-10)(y-2)=0$ $\Rightarrow x^2-8x+7+y^2-12y+20=0$ $\Rightarrow (x-4)^2+(y-6)^2=25$ **isw**			
	(iii)		B lies on circle as $(8-4)^2+(9-6)^2=16+9=25$	B1 [1]		Working must be convincing
	(iv)		gradient of AM = $\left(\frac{10-6}{1-4}\right)=\frac{4}{-3}$ gradient of BM = $\left(\frac{9-6}{8-4}\right)=\frac{3}{4}$ Since $\frac{4}{-3}\times\frac{3}{4}=-1$ the lines are perpendicular	B1 B1 B1 [3]	One gradient (need not be simplified) Second gradient (need not be simplified) **Demonstration** that $m_1\times m_2=-1$ is satisfied and all **working to derive gradients shown.**	Labelling does not need to be specific. **SC** Both gradients upside down or signs the wrong way round **B0 B0 B1**

Question			Answer	Marks	Guidance	
			Alternative: Use of Pythagoras **M1** 5, 5, $\sqrt{50}$ seen and used **A1** Arithmetic correct and final statement **A1**		Attempt to find all three lengths	
	(v)		B to M $= \begin{pmatrix} -4 \\ -3 \end{pmatrix} \Rightarrow$ M to D $= \begin{pmatrix} -4 \\ -3 \end{pmatrix}$ $\Rightarrow$ D is $(0,3)$	**M1** **A1, A1** **[3]**	Idea of **BM** = **MD** soi Each value	
			Alternative: Centre as midpoint: Idea **M1** $\left(\frac{8+x}{2}\right) = 4 \Rightarrow x = 0$ Each value **A1 A1** $\left(\frac{9+y}{2}\right) = 6 \Rightarrow y = 3$			
			Alternative: Equation BM is $y = \frac{3}{4}x + 3$ Sub in eqn for circle $\Rightarrow x^2 - 8x = 0$ $\Rightarrow x = 0$ Sub to give $y = 3$ Idea **M1** Each value **A1 A1**			

Question			Answer	Marks	Guidance	
11	(i)		$\frac{dy}{dx} = x$	M1	Differentiation	If no differentiation then 0/5
			At A gradient of tangent = -2	A1		
			so gradient of normal $= \frac{1}{2}$.	A1ft	Follow through their gradient of tangent.	
			$\Rightarrow$ Eqn of AB is $y - 2 = \frac{1}{2}(x+2)$	M1dep	Using (–2, 2) and their **normal** gradient	
			$\Rightarrow 2y = x + 6$ **oe**	A1 [5]	3 terms only	
	(ii)		line meets curve when $x^2 = x + 6$	M1	Equate *their* straight line to given curve.	
			$\Rightarrow x^2 - x - 6 = 0$	A1	Quadratic	
			$\Rightarrow (x-3)(x+2) = 0$			
			$\Rightarrow$ At B $x = 3,\ y = \frac{9}{2}$	A1 [3]		
	(iii)		Area between = Area under line – area under curve	M1	Attempt to evaluate area under curve by integration **soi**	Seen by power increased by 1. Care not to multiply by *x*
				M1	Attempt to evaluate area under their straight line by trapezium or integration **soi**	*Ignore absence of limits for first 3 marks*
			$= 16.25 - 5.833 = 10.4$	M1dep	Subtracting areas, dep on **both** M marks	
			$= 10\frac{5}{12}$	A1 [4]	Answer	

Question			Answer	Marks	Guidance	
12	(i)		Substitute: 75 = 900a + 30b 240 = 3600a + 60b Solve: $\Rightarrow a=\frac{1}{20}, b=1 \Rightarrow d=\frac{1}{20}v^2+v$	B1 B1 M1 A1 A1 **[5]**	Allow unsimplified coefficients Solve *a* *b*	ie equal coefficients **and** subtract or correct substitution. NB Answers given so algebra for first value found must be convincing.
	(ii)		$D=\left(\frac{4900}{20}+70\right)-\left(\frac{4225}{20}+65\right)$ $=38.75$	M1 A1 A1 **[3]**	Calculation at each value and subtraction attempted For either 315 or 276.25 **soi** Allow 38.8	 Or 33.75 or 5
	(iii)		Substitute: $50=\frac{1}{20}v^2+v$ or $v^2+20v-1000=0$ $\Rightarrow v=\frac{-20\pm\sqrt{400+4000}}{2}$ ≈ 23.2 mph	M1 A1 M1 A1 **[4]**	Substitute Quadratic (in any form) **isw** Solving their quadratic using **correct** formula or completion of square **M1 A1** or **B2** answer with no working	 Correct application of completion of square is $(v+10)^2=k$ seen **SCM1** for trial and improvement with values between 20 and 25. **A1** ans correct to 3 sf **SCB2** If answer given with no quadratic. Final answer is anything that rounds to 23.2 *Ignore negative values*

Question			Answer	Marks	Guidance	
13	(i)		$(2+h)^3 = 8+3.4h+3.2h^2+h^3$ $= (8+)12h+6h^2+h^3$	B1 B1 B1 [3]	For each coefficient or term that is correct Ignore incorrect identification of coefficients after expansion Mark final line	ie allow answer left in simplified expansion form.
	(ii)		$(2 + h)^3 - 2^3$ $(2 + h) - 2$ Gradient = $\dfrac{(2+h)^3-8}{2+h-2} = \dfrac{(2+h)^3-8}{h}$	B1 B1 B1 [3]	Change in *y* Change in *x* Only award if you are satisfied that the algebra is correct	Accept description in words
	(iii)		$\dfrac{(2+h)^3-8}{h} = \dfrac{8+12h+6h^2+h^3-8}{h}$ $= \dfrac{12h+6h^2+h^3}{h} = 12+6h+h^2$	M1 A1 [2]	Or using their part **(i)**	
	(iv)		*Their* 12 in (iii)	B1 [1]	Dependent on **(iii)** being a polynomial.	This answer must be consistent with **(iii)**
	(v)		$(2+h)^4 = 16+32h+24h^2+8h^3+h^4$ Gradient of chord = $32 + 24h + 8h^2 + h^3$ Giving 32**www**	B1 B1 B1 [3]	Allow 16 + 32*h* + (higher orders of *h*) Allow 32 + (higher orders of *h*) Dependent on previous work	

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OCR
RECOGNISING ACHIEVEMENT

Wednesday 5 June 2013 – Afternoon

FSMQ ADVANCED LEVEL

6993/01 Additional Mathematics

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 6993

Other materials required:
- Scientific or graphical calculator

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

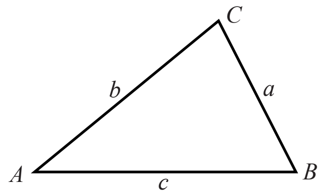
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

DC (CW/SW) 55777/3

Formulae Sheet: 6993 Additional Mathematics

In any triangle *ABC*

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$



Binomial expansion

When n is a positive integer

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Section A

1 **(i)** Find the gradient of the line, L, whose equation is $3x + 2y = 7$. **[2]**

(ii) Find the equation of the line which is perpendicular to L and which passes through the point (3, 1). **[3]**

2 Find the integers that satisfy the inequality $-7 < 3x + 1 < 12$. **[4]**

3 This year John is 4 times as old as his son Paul. In 5 years' time John will be only 3 times as old as Paul.

Let the age of Paul now be x years.

By forming an equation in x and solving it, find Paul's age now. **[4]**

4 You are given that θ is an acute angle and $\sin\theta = \frac{\sqrt{5}}{3}$.

Find the **exact** value of $\tan\theta$. **[3]**

5 **(i)** Use calculus to find the stationary points on the curve $y = x^3 - \frac{3}{2}x^2 - 6x + 3$. **[5]**

(ii) Sketch the curve on the axes provided showing the stationary points and the point where it cuts the y-axis. **[2]**

6 Amanda throws 3 fair dice. What is the probability that

(i) exactly 2 sixes are thrown, **[3]**

(ii) at least 1 six is thrown? **[3]**

7 John and Jennie are asked to draw a triangle ABC with the following properties:

AC = 6 cm, CB = 4 cm and the angle A = 40°.

John draws the triangle as shown in Fig. 7.1 and Jennie draws the triangle as shown in Fig. 7.2.

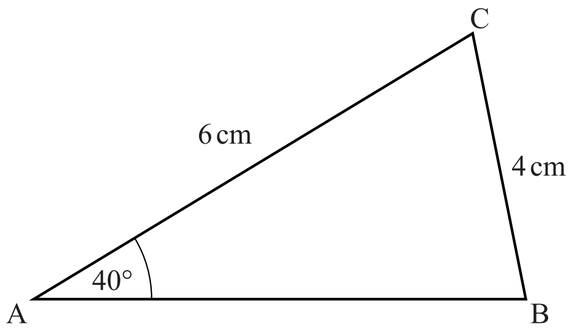


Fig. 7.1

Not to scale

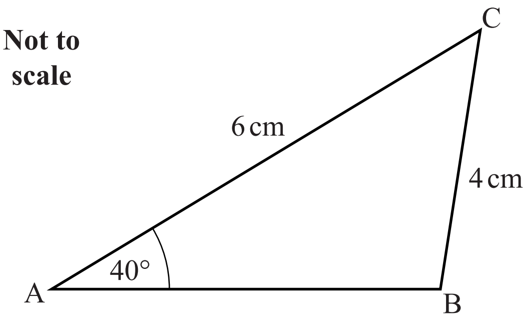
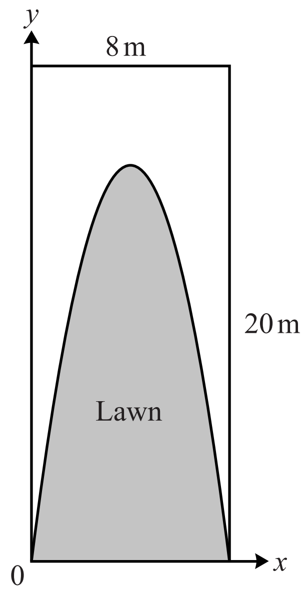


Fig. 7.2

Calculate the angle B in each case. **[4]**

8 A mathematical gardener has a garden which is rectangular in shape measuring 20 metres by 8 metres. He wishes to arrange the garden so that approximately half of it is lawn and the rest flower bed.

He sets up a coordinate system as shown in the diagram below and maps out the graph of the curve $y = 8x - x^2$.



Show that the area of the lawn is approximately 53% of the total area. **[6]**

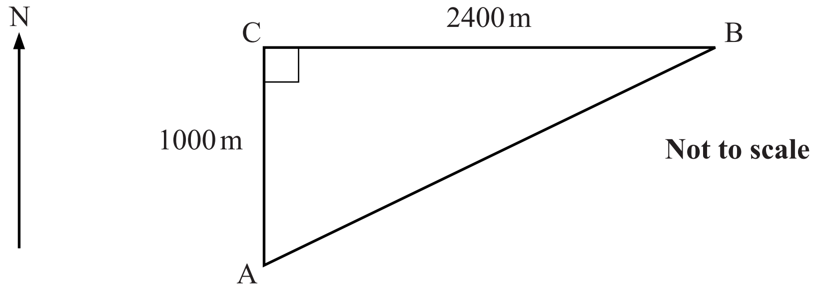
9 **(i)** Find the values of the constants a and b such that, for all values of x

$$x^2 + 8x + 19 = (x + a)^2 + b.$$ **[3]**

(ii) Hence state the least value of $x^2 + 8x + 19$ and the value of x at which this occurs. **[2]**

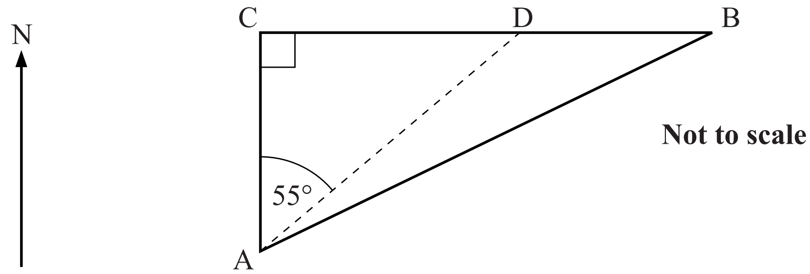
(iii) Write down the greatest value of $\dfrac{1}{x^2 + 8x + 19}$. **[1]**

10 One leg of a cross-country race is from A to B. The checkpoint B is at the end of a wall that runs due east-west, as shown in the diagram. A is a point 1000 m due south of a point C on the wall. BC = 2400 m.



(i) What bearing should a runner take to travel from A to B and what is the distance AB? **[4]**

John sets off from A unable to see the checkpoint, B. He heads out on a bearing of 055° and when he reaches the wall at point D he knows he has to go east along the wall to reach the point B, as shown in the diagram.



(ii) How much further than the distance AB does John run? **[3]**

Section B

11 A circle has equation $(x-2)^2 + y^2 = 100$.

(a) Write down the radius and the coordinates of the centre, C, of this circle. **[2]**

The line $y = 2x + 6$ cuts the circle at two points, A and B.

(b) Find

(i) the coordinates of A and B, **[5]**

(ii) the midpoint, M, of AB, **[1]**

(iii) the length AB. **[2]**

(c) Hence find the distance of the centre of the circle from the line AB. **[2]**

12 An object sinks through a thick liquid such that at time t seconds after being released on the surface the depth, s metres, is given by

$$s = 4t^2 - \frac{2t^3}{3} \qquad \text{for } 0 \leqslant t \leqslant 4.$$

(a) Find the formula for the velocity, v metres per second, t seconds after being released. Hence show that the object stops sinking when $t = 4$. **[4]**

(b) Find

(i) the acceleration of the object when it is released on the surface of the liquid, **[4]**

(ii) the greatest depth of the object. **[2]**

(c) On the grids provided sketch the velocity-time and acceleration-time graphs. **[2]**

13 A number of students from a group of 20 boys and 30 girls are to be selected to attend a one-day conference.

The number of girls attending must be at least the same as the number of boys but no more than twice the number of boys.

(i) Let there be x boys and y girls selected. Given that $x > 0$ and $y > 0$, write down four more inequalities to represent the information. **[3]**

(ii) Plot these inequalities on the grid provided. Indicate the region for which the inequalities hold. Shade the area that is **not** required. **[5]**

(iii) In order to attend the conference the students need to be given a special uniform. The uniform for the boys costs £40 and the uniform for the girls cost £50. The school has £2000 to spend on the uniforms.

By plotting the appropriate line on your graph, find the maximum number of students that could go to the conference. **[4]**

14 A curve has equation $y = 4x^3 - 5x^2 + 1$ and passes through the point A(1, 0).

(i) Find the equation of the normal to the curve at A. **[5]**

(ii) This normal also cuts the curve in two other points, B and C. Show that the x-coordinates of the three points where the normal cuts the curve are given by the equation $8x^3 - 10x^2 + x + 1 = 0$. **[2]**

(iii) Show that the point B $\left(\frac{1}{2}, \frac{1}{4}\right)$ satisfies the normal and the curve. **[2]**

(iv) Find the coordinates of C. **[3]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.

RECOGNISING ACHIEVEMENT

FSMQ

Additional FSMQ

Free Standing Mathematics Qualification

6993: Additional Mathematics

Mark Scheme for June 2013

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations

Annotation	Meaning
✓	Tick
✗	Cross
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0	Method mark awarded 0
M1	Method mark awarded 1
A0	Accuracy mark awarded zero
A1	Accuracy mark awarded 1
B0	Independent mark zero
B1	Independent mark 1
SC	Special case
^	Omission mark
MR	Misread

Subject-specific Marking Instructions

1 M (method) marks are not lost for purely numerical errors.
A (accuracy) marks depend on preceding M (method) marks. Therefore M0 A1 cannot be awarded.
B (independent) marks are independent of M (method) marks and are awarded for a correct final answer or a correct intermediate stage.

2 Subject to 2, two situations may be indicated on the mark scheme conditioning the award of A marks or independent marks:
i. Correct answer correctly obtained (no symbol)
ii. Follows correctly from a previous answer whether correct or not (“” on mark scheme and on the annotations tool).

3 Always mark the greatest number of significant figures seen, even if this is then rounded or truncated in the answer.

4 Where there is clear evidence of a misread, a penalty of 1 mark is generally appropriate. This may be achieved by awarding M marks but not an A mark, or awarding one mark less than the maximum.

5 Where a follow through () mark is indicated on the mark scheme for a particular part question, you must ensure that you refer back to the answer of the previous part question if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

Abbreviations

The following abbreviations are commonly found in Mathematics mark schemes.

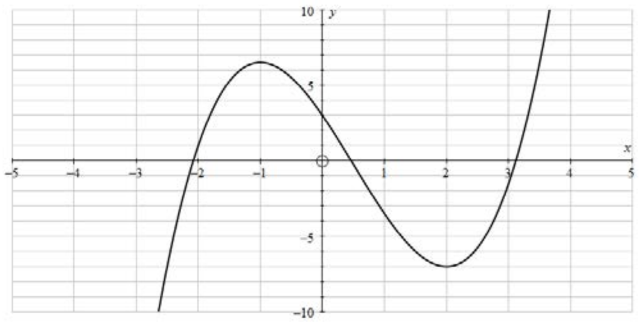
- Where you see **oe** in the mark scheme it means **or equivalent;**
- Where you see **cao** in the mark scheme it means **correct answer only;**
- Where you see **soi** in the mark scheme it means **seen or implied;**
- Where you see **www** in the mark scheme it means **without wrong working;**
- Where you see **rot** in the mark scheme it means **rounded or truncated;**
- Where you see **seen** in the mark scheme it means that you should award the mark if that number/expression is seen anywhere in the answer space, even if it is not in the method leading to the final answer;
- Where you see **figs 237**, for example, this means any answer with only these digits. You should ignore leading or trailing zeros and any decimal point eg 237000, 2·37, 2·370, 0·00237 would be acceptable but 23070 or 2374 would not.

Section A

Question			Answer	Marks	Rationale
1	(i)		Gradient = $^-1.5$	**M1** **A1** **2**	Attempt to rearrange to to $y =$ ..... Mark final answer (Ans $= {}^-1.5x$ is **M1A0**)
			Alternative method: Find two points on line and then the gradient by $\frac{\Delta y}{\Delta x}$ **M1** (The two points **must** lie on the line) Ans **A1**		
1	(ii)		Gradient = $\frac{2}{3}$ Use (3,1) and *their* normal gradient in a standard form for a line $\Rightarrow 3y = 2x - 3$ **oe**	**M1** **M1** **A1** **3**	**soi** There must be an attempt to find a normal gradient for this method mark to be earned 3 terms only
			Alternative method: $2x - 3y = k$ **B1** Sub (3,1) **M1** Ans **A1**		

Question			Answer	Marks	Rationale
2			$-7 < 3x + 1 < 12$ $\Rightarrow -8 < 3x < 11$ $\Rightarrow -\frac{8}{3} < x < \frac{11}{3}$ $\Rightarrow -2, -1, 0, 1, 2, 3$	M1 M1 A1 A1 4	Subtract either side by 1 (or $\frac{1}{3}$ if done the other way round) Divide throughout by 3 Or $-\frac{8}{3} < x$ and x$<\frac{11}{3}$ **Mark final answer** **SC1** for $x < \frac{11}{3}$ or $x > -\frac{8}{3}$ but not both
			Alternative method: Can be done by trial: **B4** all correct One missing or one extra **B2** **Mark final answer**		
3			John's age, 4x **soi** Ages in 5 yrs x + 5, 4x + 5 **soi** 4x + 5 = 3(x + 5) 4x + 5 = 3x + 15 x = 10 No ISW	B1 B1 M1 A1 4	Condone use of their own letters **SC2** Answer only with no equation formed or incorrect algebra
			Alternative method: Forming simultaneous equations: Correct equations implies **B1**, **B1** Eg j = 4p and j + 5 = 3(p + 5) Give **M1** only if there is an attempt to eliminate one variable.		

Question			Answer	Marks	Rationale
4			$\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{5}{9} = \frac{4}{9}$	**M1**	**Any use of calculators to approximate gets A0**
			$\Rightarrow \cos\theta = \frac{2}{3}$	**A1**	
			$\Rightarrow \tan\theta = \frac{1}{2}\sqrt{5}$ ISW	**A1**	
				3	
			Alternative method: Find third side of triangle **M1** **= 2** **A1** Ans **A1**		

Question			Answer	Marks	Rationale
5	(i)		$y = x^3 - \frac{3}{2}x^2 - 6x + 3$	M1	Diffn – all powers reduced by 1 – allow one error (beware dividing by *x*)
			$\Rightarrow \frac{dy}{dx} = 3x^2 - 3x - 6$	A1	All terms correct
			$\Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$	M1	Set = 0 and solve (dependent on first M mark)
			$\Rightarrow x = 2, -1$	A1	Both *x*
			$\Rightarrow y = -7, 6.5$		
			$\Rightarrow (2,-7),\ (-1,6.5)$	A1	Both pairs (Allow this mark if the coordinated are explicitly stated in 5**(ii)**.)
				5	
5	(ii)			B1	Correct shape (Cubic the right way up and two turning points, does not need three intercepts on *x*-axis)
				B1	Through (0, 3), (2, –7) and (–1, 6.5). Dep on 1st B mark Allow *y*-intercept in range [2,4] At *x* = 2 allow *y* in range [–6, –8] At *x* = –1 allow *y* in range[6, 8]
				2	

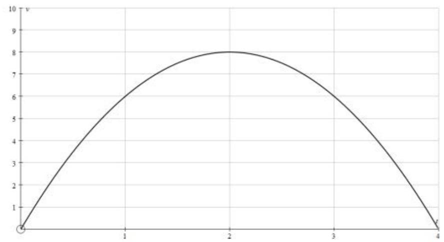
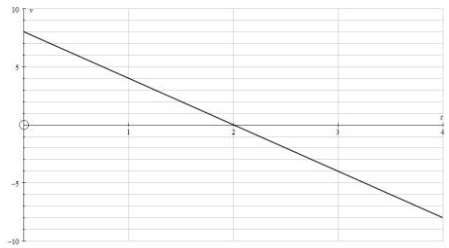
Question			Answer	Marks	Rationale
6	(i)		P(2 from 3) $=\binom{3}{2}p^2q$	**B1**	1 term with correct fractions and powers
			$=3\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)$	**B1**	Correct coefficient of 3 attached to their probability term **soi**
			$=\frac{5}{72}$ or 0.0694... **oe** ISW	**B1**	At least 3 sf Correct answer only **B3**
				3	
6	(ii)		P(at least 1) $=1-\text{P}(0)=1-q^3$	**M1**	
			$=1-\left(\frac{5}{6}\right)^3$	**A1**	
			$=\frac{91}{216}$ or 0.421...	**A1**	At least 3 sf
				3	
			Alternative method: P(1) + P(2) + P(3) **M1** Add three terms = 0.3472 + 0.0694 + 0.0046 **A1** Three terms correct **soi** = 0.421… **A1**		
7			Sin rule: $\frac{\sin 40}{4}=\frac{\sin B}{6}$	**M1**	Sin rule or complete method via the perpendicular to find angle
			$\Rightarrow \sin B=\frac{6\times\sin 40}{4}(=0.964....)$	**A1**	Correctly applied in this case **soi**
			$\Rightarrow B=74.6\ (^\circ)$ or 105.4	**A1**	One value
			and $105(.4^\circ)$ or 74.6	**B1**	**FT** 180 – *their B*, unless *B* = 90
				4	

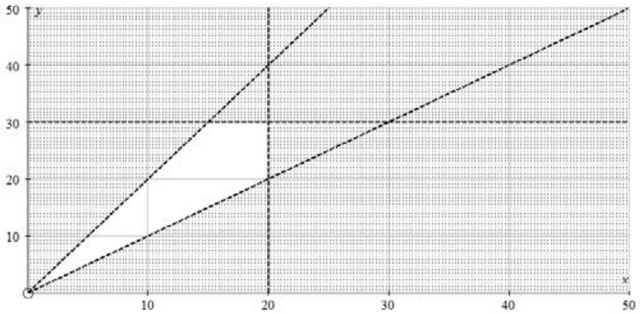
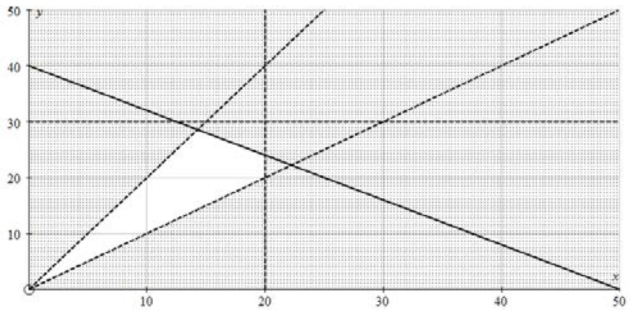
Question			Answer	Marks	Rationale
8			$\int_0^8 (8x - x^2)\,dx = \left[4x^2 - \frac{x^3}{3}\right]_0^8$	M1 A1	Increase in power of 1 in at least one term (beware multiplying by x) Both terms
			$= 4\times 8^2 - \frac{8^3}{3} = 64\times\frac{4}{3}$	M1	Substitute limits of 0 and their upper limit following integration
			$= \frac{256}{3}$ or $85\frac{1}{3}$ or 85.3 or better	A1	(NB Limits the wrong way round **M1 A0**)
			Total area = 160 $\Rightarrow$ Proportion = $\frac{\frac{256}{3}}{160} = \frac{256}{480} = 53.3\%$	M1 A1 6	Dep on previous M, ratio of their ans / 160 Dep on previous A1 (NOT 53) NB The answer is given.
			Alternative method: 53% of 160 **M1** dep on all other M marks = 84.8 The two answers related, eg both corrected to 2 sf = 85 **A1**		Dep on previous **A1**
9	(i)		$x^2 + 8x + 19 = (x+4)^2 + 19 - 16$ $= (x+4)^2 + 3$	M1 A1 A1 3	Attempt to complete square or expand rhs to give quadratic expression For 4 **www** For 3 NB For completion of square at least $(x \pm 4)^2$ seen
9	(ii)		When $x = -$ *their a* Value is *their b*	B1 B1 2	**FT** **FT**
9	(iii)		$\frac{1}{\textit{their } b}$	B1 1	**FT**

Question			Answer	Marks	Rationale
10	(i)		Angle CAB = $\tan^{-1}\frac{2400}{1000} = 67.4°$ or bearing = 067^0 Distance = $\sqrt{1000^2 + 2400^2} = 2600$ (Rounds to 2600 to 4 sf)	**M1** **A1** **M1** **A1** **4**	Mark final answer (Don't allow 67) Can be awarded if seen in part **(ii)**
10	(ii)		AD = $\frac{1000}{\cos 55}$ or anything that rounds to 1743 CD = $1000\tan 55$ or anything that rounds to 1428 DB = 2400 – CD = 972 $AD + DB - AB =$ anything that rounds to 115(m)	B1 **B1** **B1** **3**	Don't accept premature approximation **cao www** Correct answer **www B3**
			Alternative method: Sine rule on right-hand triangle $\frac{x}{\sin 22.6} = \frac{y}{\sin 12.4} = \frac{2600}{\sin 145}$ **B1** (all values seen, angles rounding to 12.4 and 22.6) $x = 1742 - 1744$ $y = 971 - 973$ **B1** both values Answer **B1**		Accept a combination

Section B

Question			Answer	Marks	Rationale
11	**(a)**		Radius 10	**B1**	
			Centre (2, 0)	**B1**	
11	**(b)**	**(i)**	Substitute $y=2x+6$ into $(x-2)^2+y^2=100$ $\Rightarrow (x-2)^2+(2x+6)^2=100$ $\Rightarrow 5x^2+20x+40=100$ $\Rightarrow x^2+4x-12=0$ $\Rightarrow (x+6)(x-2)=0$ $\Rightarrow$ A is $(-6,-6)$ and B is $(2,10)$	**M1** **A1** **M1** **A1** **A1** **5**	Substitute 3 term quadratic Solve a 3 term quadratic Either both *x* or both *y* or one pair Both pairs
11	**(b)**	**(ii)**	Midpoint AB = $\left(\frac{-6+2}{2},\frac{-6+10}{2}\right)\Rightarrow(-2,2)$	**B1** **1**	
11	**(b)**	**(iii)**	AB = $\sqrt{(-6-2)^2+(-6-10)^2}$ $=\sqrt{8^2+16^2}=\sqrt{320}=8\sqrt{5}$ or 17.9	**M1** **A1** **2**	
11	**(c)**		distance = $\sqrt{\text{radius}^2-\text{ half their }\textbf{(iii)}^2}$ $=\sqrt{100-80}=\sqrt{20}=2\sqrt{5}$ or 4.47	**M1** **A1** **2**	
			Alternative method: Their centre to their midpoint $=\sqrt{(2--2)^2+2^2}=\sqrt{20}$ **M1A1**		

Question			Answer	Marks	Rationale
12	(a)		$v = 8t - 2t^2$ $v = 0$ when $8t = 2t^2 \Rightarrow t = 4$ or when $t = 4$, $v = 8 \times 4 - 2 \times 4^2 = 32 - 32 = 0$	**M1** **A1** **M1** **A1** **4**	Diffn *v* Final answer Set = 0 or *t*=4, dep on first **M1** **SC3** for confirming = 0 when *v* divided by 2
12	(b)	(i)	$v = 8t - 2t^2$ $\Rightarrow a = 8 - 4t$ When $t = 0$, $a = 8$	**M1** **A1** **M1** **A1** **4**	Diffn *their v* Set $t = 0$ dep on 1st **M1**
12	(b)	(ii)	$t = 4$ gives $s = 64 - \frac{128}{3} = 21.3...$	**M1** **A1** **2**	Substitute *t*=4 **oe**
12	(c)		 	**B1** **B1** **2**	Inverted parabola implies symmetry about t = 2 through (0,0) and (4,0) (sight of correct vertex not necessary) Straight line through (2, 0) and (0, *their* 8) (4, –*their* 8)

Question			Answer	Marks	Rationale
13		(i)	$x \le 20, y \le 30$ $y \ge x$ $y \le 2x$	**B1** **B1** **B1** **3**	Condone < for ≤ and > for ≥ Ignore extras (including for instance, $x + y \le 50$)
13		(ii)		**B1** **B1** **B1** **B1** **B1** **5**	$x = 20$ $y = 30$ $y = x$ $y = 2x$ Shading Ignore any shading that may relate to 13**(iii)**
13		(iii)	The line is $40x + 50y = 2000$  (20, 24) Total = 44 **www**	**B1** **B1** **B1** **B1** **4**	accept inequality **soi** (accept written in **(ii)**) Draw line (Correct line seen on graph is **B2**) Point **soi** Total (Total of 44 seen with nothing else is **B2**)

Question			Answer	Marks	Rationale
14		(i)	$\frac{dy}{dx}=12x^2-10x$ When $x=1$, $m=2$ $\Rightarrow$ gradient of normal $=-\frac{1}{2}$ $\Rightarrow$ Equation is $y-0=-\frac{1}{2}(x-1)$ $\Rightarrow 2y+x=1$	M1 A1 A1 M1 A1 5	Diffn and sub $x = 1$ **www** **FT** *their* numerical m **soi** Find eqn of line dep on M1 and *their* **normal** gradient **oe** $\left(\text{eg } y=-\frac{1}{2}x-\frac{1}{2}\right)$
14		(ii)	$-\frac{1}{2}(x-1)=4x^3-5x^2+1$ $\Rightarrow 1-x=8x^3-10x^2+2$ $\Rightarrow 8x^3-10x^2+x+1=0$	M1 A1 2	Substitute *their equation* At least 1 correct intermediate step seen, beware answer given
14		(iii)	For line: $y=-\frac{1}{2}\left(\frac{1}{4}-1\right)=-\frac{1}{2}\times-\frac{1}{2}=\frac{1}{4}$ **oe** For curve: $y=4\left(\frac{1}{2}\right)^3-5\left(\frac{1}{2}\right)^2+1=\frac{1}{2}-\frac{5}{4}+1=\frac{1}{4}$	B1 B1 2	Substitute into *correct* line Substitute into curve

Question			Answer	Marks	Rationale
14		(iv)	$f(x)=0$ when $x=1$ and $\frac{1}{2}$ $\Rightarrow f(x)=(x-1)(2x-1)(4x+1)$ $\Rightarrow x=-\frac{1}{4}$ $y=\frac{5}{8}$ $\Rightarrow$ C is $\left(-\frac{1}{4},\frac{5}{8}\right)$ **www**	M1 A1 A1 3	$f(x) = (x - 1)(2x - 1)(ax + b)$ **oe** or $f(x) = (x - 1)(x - \frac{1}{2})(cx + d)$ NB The working for this part may appear elsewhere but can only be credited if their final answer is seen here.
			Alternative method: Long division by $(x–1)$ and $(2x–1)$ or by one given factor **plus** attempt to factorise resulting quadratic or by $(2x^2–3x+1)$ **M1** Giving third root **A1** Answer **A1**		Evidence of long division on cubic is correct first line of long division plus kx^2 in the quotient

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OCR

Friday 6 June 2014 – Afternoon

FSMQ ADVANCED LEVEL

6993/01 Additional Mathematics

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 6993/01

Other materials required:
- Scientific or graphical calculator

Duration: 2 hours

1264621551

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

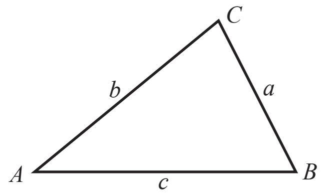
This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Formulae Sheet: 6993 Additional Mathematics



In any triangle *ABC*

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Binomial expansion

When n is a positive integer

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Section A

1 Solve the following.

$$-6 < 2x - 1 < 7$$ **[3]**

2 The gradient function of a curve that passes through the point (1, 2) is given by

$$\frac{dy}{dx} = 3x^2 - 4x + 7.$$

Find the equation of the curve. **[4]**

3 **(i)** Find the area enclosed between the curve $y = 8x^3$, the x-axis and the line $x = 2$. **[3]**

(ii) Hence, or otherwise, deduce the area between the x-axis, the y-axis, the line $x = 2$ and the curve $y = 8x^3 + 5$. **[1]**

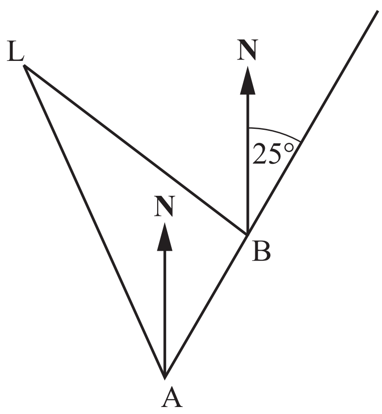
4 A train travels from station A to station B. It starts from rest at A and comes to rest again at B. The displacement of the train from A at time t seconds after starting from A is s metres where

$$s = 0.09t^2 - 0.0001t^3.$$

(i) Find the velocity at time t seconds after leaving A and hence find the time taken to reach B. Give the units of your answer. **[4]**

(ii) Find the distance between A and B. Give the units of your answer. **[2]**

5 A ship is moving on a bearing of 025º at 14 knots (1 knot = 1 nautical mile per hour). As it passes point A, a lighthouse L is seen on a bearing of 340º. After 30 minutes, the ship passes point B from where the lighthouse is seen on a bearing of 320º.



Not to scale

(i) Find the angle BAL and the angle ALB. **[3]**

(ii) Hence, or otherwise, calculate the distance BL in nautical miles. **[3]**

6 The function $f(x) = x^3 - 4x^2 + ax + b$ is such that

- $x = 3$ is a root of the equation $f(x) = 0$,
- when $f(x)$ is divided by $(x - 1)$ there is a remainder of 4.

(i) Find the value of a and the value of b. **[4]**

(ii) Solve the equation $f(x) = 0$. **[3]**

7 The points A and B have coordinates (3, 7) and (5, 11) respectively.

(i) Find the exact length of AB. **[2]**

(ii) Find the equation of the circle with diameter AB. **[3]**

8 Four points have coordinates A(−5, −1), B(0, 4), C(7, 3) and D(2, −2).

(i) Using gradients of lines, prove that ABCD is a parallelogram. **[2]**

(ii) Using lengths of lines, prove further that ABCD is a rhombus. **[2]**

(iii) Prove that ABCD is not a square. **[2]**

9 **(i)** Show that $\frac{1-\cos^2 x}{1-\sin^2 x} = \tan^2 x$. **[1]**

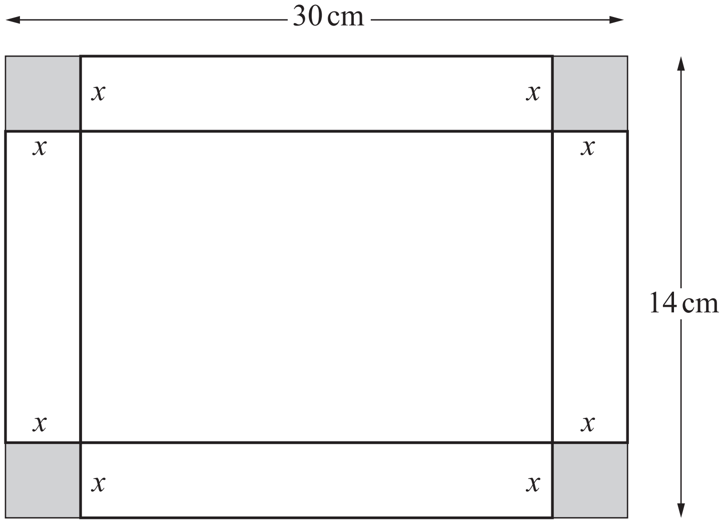
(ii) Hence solve the equation $\frac{1-\cos^2 x}{1-\sin^2 x} = 3 - 2\tan x$ for values of x in the range $0° \leqslant x \leqslant 180°$. **[4]**

10 **(i)** Find the coordinates of the point P on the curve $y = 2x^2 + x - 5$ where the gradient of the curve is 5. **[3]**

(ii) Find the equation of the normal to the curve at the point P. **[3]**

Section B

11 Kala is making an open box out of a rectangular piece of card measuring 30 cm by 14 cm. She cuts squares of side x cm out of each corner and turns up the sides to form the box.



(i) Find an expression in terms of x for the volume, V cm^3, of the box and show that this reduces to

$V = 4x^3 - 88x^2 + 420x$. **[4]**

(ii) Find the two values of x that give $\frac{dV}{dx} = 0$. **[5]**

(iii) Explain why one of these values should be rejected and find the maximum volume of the box using the other value. **[3]**

12 Paul walked from Anytown to Nexttown, a distance of 15 km. When he got there he then walked back. His average speed on the return journey was 2 km per hour less than on the outward journey.

Let Paul's average speed on the outward journey be x km hr^{-1}.

(i) Write down an expression for the time, in hours, taken for the whole journey. **[2]**

The time taken by Paul for the whole journey was 6 hours.

(ii) Use your expression in **(i)** to form an equation in x and show that it simplifies to

$$x^2 - 7x + 5 = 0.$$ **[4]**

(iii) Solve this equation to find Paul's average speed on the outward journey. **[3]**

(iv) Find the difference in time between the outward and return journeys. Give your answer to the nearest minute. **[3]**

13 A company needs to buy some storage units. There are two types of unit available, type X and type Y. The cost of each type of unit, the floor space required and the volume for storage are given in the following table.

	Cost per unit (£)	**Floor space required (m^2)**	**Volume for storage (m^3)**
X	100	2	3.5
Y	120	1.5	3

The maximum cost allowed for the purchase of the units is £1200 and the maximum floor space available is $18\,m^2$.
The company wants to maximise the volume for storage.

Let x and y be the number of each type of unit, X and Y, respectively.

(i) Write down an inequality for the total cost and an inequality for the total floor space required. **[3]**

(ii) Draw the inequalities you gave in **(i)** on the grid provided in the answer book. Given that $x \geqslant 0$ and $y \geqslant 0$, indicate the region for which the inequalities hold by shading the area that is **not** required. **[4]**

(iii) Write down the objective function for the volume for storage and find the combination of units that should be bought to maximise the volume for storage.
Write down this maximum volume. **[5]**

14 Mugs are packed in boxes of 10. On average, 5% of the mugs are imperfect. A box of mugs is classified as "unsatisfactory" if it contains two or more imperfect mugs.

(i) State two conditions that must be satisfied for the number of imperfect mugs in a box to have a binomial distribution. **[2]**

(ii) Assuming that these two conditions are satisfied, calculate the probability that a box chosen at random is "unsatisfactory". **[6]**

A shop receives a delivery of a large number of boxes of mugs. The delivery is checked as follows.

A box is chosen at random.
- If there are no imperfect mugs in the box then the whole delivery is accepted.
- If the box is "unsatisfactory" then the whole delivery is rejected.
- If there is exactly one imperfect mug in the box then a second box is chosen at random. The delivery is accepted only if this box contains no imperfect mugs.

(iii) Calculate the probability that the delivery is accepted. **[4]**

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THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.

Oxford Cambridge and RSA

FSMQ

Additional Mathematics

Unit **6993:** Additional Mathematics

Free Standing Mathematics Qualification

Mark Scheme for June 2014

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for 6993

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation **isw**. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument. Unless otherwise stated (by for instance, **cao** usually apply **isw**

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation **ft** implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Section A

Question			Answer	Marks	Guidance	
1			$-6 < 2x - 1 < 7$ $\Rightarrow -5 < 2x < 8 \Rightarrow -\frac{5}{2} < x < 4$ **isw** **S.C**. If solution only given as number line with two clear open circles at ends then B2	**B1** **B1** **B1** **[3]**	One end point The other Both together - accept 2 separate inequalities linked by "and" **cao** **B3** Final answer seen **www**	Condone incorrect signs (including equals) for first 2 marks. Ignore listings. but not "or" or comma or nothing

Question			Answer	Marks	Guidance	
2			$\frac{dy}{dx} = 3x^2 - 4x + 7$ $\Rightarrow y = x^3 - 2x^2 + 7x + c$ **oe** Substitute (1, 2) $\Rightarrow 2 = 1 - 2 + 7 + c$ $\Rightarrow c = -4$ $\Rightarrow y = x^3 - 2x^2 + 7x - 4$	**M1** **A1** **M1** **A1** **[4]**	Integrate (at least 2 powers increased by 1) Ignore lack of y = and c Dependent on 1st M Final equation must be seen	Beware just multiplying by x Must include y = and –4 Condone f(x) =

Question			Answer	Marks	Guidance	
3	(i)		Area = $\int_0^2 8x^3\,dx = \left[2x^4\right]_0^2$	M1	Integrate correct function (power of 4 **soi**). Ignore wrong or no limits	Beware just multiplying by x i.e. $8x^4$
				M1	Substitute $x = 2$. Dependent on first M	
			$= 32 \ (-0) = 32$	A1 [3]		
	(ii)		Add a rectangle of area 10 $\Rightarrow$ Total area = 42	B1 [1]	**ft** Addition of 10 must be correct!	Can be by integration

Question			Answer	Marks	Guidance	
4	(i)		$s = 0.09t^2 - 0.0001t^3$ $\Rightarrow$ $v = 0.18t - 0.0003t^2$ **isw**	M1 A1	Diffn (both powers reduced by 1).	Beware division by t.
			When $v = 0$, $t = 0$ or $t = \frac{0.18}{0.0003} = 600$	M1	Set $v = 0$ and any attempt to solve. Dependent on first M.	Condone division by t or a constant when expression not set to 0
			Time = 600 seconds or 10 mins **isw**	A1 [4]	Units required.	
	(ii)		Substitute *their* t into s $\Rightarrow s$ = 10800 m or 10.8 km **isw**	M1 A1 [2]	Units required, but only withhold this mark for units wrong or missing if not already withheld in part **(i)**.	

Question			Answer	Marks	Guidance	
5	(i)		Angle BAL = 20 + 25 = 45	B1		BAL and ALB must be correctly identified (not from use in (ii))
			Angle ABL = 180 – 65 = 115 **soi** OR exterior angle = 65 **soi** OR angle LBN = 40 **soi**	B1		
			Angle ALB = 20	B1 [3]	Or **B2** for ALB **www**	
	(ii)		AB = 7 **soi**	B1		
			$\frac{LB}{\sin 45} = \frac{AB}{\sin 20}\left(= \frac{7}{\sin 20}\right)$	M1	For *their* AB and *their* angles.	
			$\Rightarrow LB = \frac{7\sin 45}{\sin 20} = 14.5$	A1 [3]	Anything that rounds to 14.5	

Question			Answer	Marks	Guidance	
6	(i)		f(3) = 0 $\Rightarrow$ 27 – 36 + 3a + b = 0 or better	B1	e.g. 3a + b = 9.	Powers need to be evaluated
			f(1) = 4 $\Rightarrow$ 1 – 4 + a + b = 4 or better	B1	e.g. a+ b = 7	Powers need to be evaluated
			Solve *their* simultaneous eqns from above	M1	Attempt to find a and b from *their* eqns	Their working need not be correct
			$\Rightarrow a = 1, b = 6$	A1 [4]		
	(ii)		Sight of $(x – 3)(x^2 + px + q)$ for any p, q Or algebraic division	M1	Or attempt to find another root of *their cubic* by remainder theorem **soi**	Algebraic division seen by x^2 + ... in quotient and $x^3 – 3x^2$ in division
			$\Rightarrow (x – 3)(x + 1)(x – 2) = 0$	A1	For correct complete factorisation **soi** by final answer	
			$\Rightarrow (x =)\ 3, 2, –1$	A1 [3]	Correct solution	

Question			Answer	Marks	Guidance	
7	(i)		$\text{Distance}^2 = (5-3)^2 + (11-7)^2 \quad (=20)$	**M1**	**Soi** e.g. by 4.47…..	d = 20 is M0
			$\Rightarrow \text{distance} = \sqrt{20} \quad = 2\sqrt{5}$	**A1**	Must be exact **isw**	
				[2]		
	(ii)		Centre = midpoint = (4, 9)	**B1**	Centre	
			Radius = $\sqrt{5}$ or decimal equivalent	**B1**	Radius **soi** by for e.g. $r^2 = 5$	
			$\Rightarrow (x-4)^2 + (y-9)^2 = 5$ OR $x^2 + y^2 - 8x - 18y + 92 = 0$	**B1**	ft *their* identified centre (but don't accept A or B) **isw**	Rhs must be 5, not $(\sqrt{5})^2$
			Alternative: $\frac{y-11}{x-5}.\frac{y-7}{x-3} = -1$	**B1**	Use of $m_1 m_2 = -1$	
			$\Rightarrow (y-11)(y-7) + (x-5)(x-3) = 0$	**B1**		
			$\Rightarrow x^2 + y^2 - 8x - 18y + 92 = 0$	**B1**		
				[3]		

Question		Answer	Marks	Guidance	
8	(i)	Grad AB = Grad CD $=1\ \left(=\frac{4--1}{0--5}\right)$ and $\left(=\frac{-2-3}{2-7}\right)$ **oe** Grad BC = Grad AD $=-\frac{1}{7}\ \left(=\frac{3-4}{7-0}\right)$ and $\left(=\frac{-2--1}{2--5}\right)$ Two pairs of parallel sides (means ABCD parallelogram)	**B1** **B1** **[2]**	For showing one pair of gradients equal and correct **www** For showing other pair of gradients equal and correct plus completion	**bod** no working but care about seeing $\frac{\delta x}{\delta y}$ Final statement is necessary. Condone 2 pairs of equal gradients (providing they are correct)
	(ii)	$AB^2=5^2+5^2$ (=50) **oe** for any side $BC^2=1^2+7^2$ (=50) $\Rightarrow AB^2=BC^2$ (=50) Equal sides (means rhombus)	**B1** **B1** **[2]**	One length (or squared length) For adjacent length plus completion **www**	Final statement is necessary
	(iii)	Gradients do not fulfil $m_1.\ m_2=-1$ **oe** ie $1\times-\frac{1}{7}\neq-1$ Therefore lines not perpendicular **Alternatives:** **A:** Use of cosine rule Does not give 90^0 **B:** Use of Pythagoras Not satisfied therefore not 90^0 **C:** Use of pythagoras to find length of diagonals (i.e. $\sqrt{160}$ and $\sqrt{40}$) Diagonals not equal	**M1** **A1** **[2]** **M1** **A1** **M1** **A1** **M1** **A1**	For use of $m_1.\ m_2=-1$ Gradients must be correct. **www** **www** **www**	i.e. 2nd gradient not the negative reciprocal of the other Final statement is necessary.

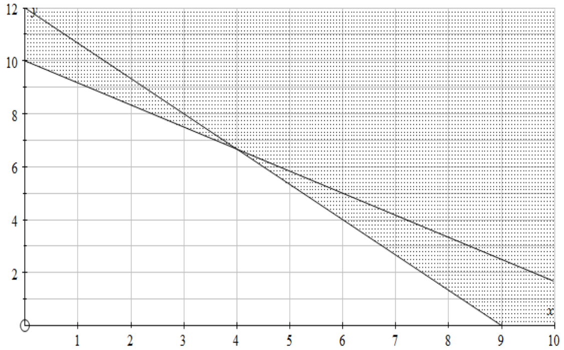
Question			Answer	Marks	Guidance	
9	(i)		$\frac{1-\cos^2 x}{1-\sin^2 x} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$	B1 [1]	Use of $\frac{\sin x}{\cos x} = \tan x$ and use of $\sin^2 x + \cos^2 x = 1$	
	(ii)		$\frac{1-\cos^2 x}{1-\sin^2 x} = 3 - 2\tan x$ $\Rightarrow \tan^2 x = 3 - 2\tan x$	M1	Correct use of **(i)**	
			$\Rightarrow \tan^2 x + 2\tan x - 3 = 0$ $\Rightarrow \tan x + 3 \quad \tan x - 1 = 0$	M1	Factorise their three term quadratic or insertion of *their* values into correct formula Dep on 1st M	(Check the two linear factors by whether they multiply out to give the first and last terms of their quadratic.)
			$\Rightarrow \tan x = -3$ or $\tan x = 1$ $\Rightarrow x = 108(.4...)$ or 45	A1A1 [4]	−1 for any other values inside range, ignore extra values outside range	

Question			Answer	Marks	Guidance	
10	(i)		$y = 2x^2 + x - 5 \Rightarrow \left(\frac{dy}{dx} =\right) \; 4x+1$	B1		
			$= 5$ $\Rightarrow x = 1, y = -2$	M1 A1 [3]	Equating to 5 and solving For both	
	(ii)		gradient normal = $-\frac{1}{5}$	B1	Gradient of normal **soi**	
			$\Rightarrow (y+2) = -\frac{1}{5} \; x - 1$	M1	Equation using *their* (1, −2) and *their* normal gradient (which may only be $\pm\frac{1}{5}$ or -5)	
			$\Rightarrow 5y + x + 9 = 0$	A1 [3]	**oe**, but only 3 terms **isw**	

Section B

Question			Answer	Marks	Guidance	
11	(i)		Length = 30 – 2x Breadth = 14 – 2x (Height = x) $\Rightarrow V = (30-2x)(14-2x)x$ $= 4x^2 - 88x + 420 \;\; x$ **oe** $= 4x^3 - 88x^2 + 420x$	B1 B1 M1 A1 [4]	**Soi** **Soi** Product of *their* length, breadth and x **www;** must show at least one product of any two lengths step (N.B. Answer given)	N.B. dimensionally correct e.g $= (30-2x)(14x-2x^2)$ Length and breadth must be functions of x
	(ii)		$V = 4x^3 - 88x^2 + 420x$ $\frac{dV}{dx} \Rightarrow 12x^2 - 176x + 420$ **isw** $= 0$ when $12x^2 - 176x + 420 = 0$ $\Rightarrow 3x^2 - 44x + 105 = 0$ $\Rightarrow \;\; 3x - 35 \;\; x - 3 \; = 0$ $\Rightarrow x = \frac{35}{3}$ or anything that rounds to 11.7, $x = 3$ **S.C.** Answers only B1, B1	M1 A1 M1 M1 A1 [5]	Diff (at least two powers reduced by 1) Set *their function* = 0 Dep on 1st M **Soi** by solution Factorise three term quadratic or insertion of *their* values into correct formula Both	Beware division by x. Condone prem div by a constant. (Check the two linear factors by whether they multiply out to give the first and last terms of their quadratic.)
	(iii)		$x = \frac{35}{3}$ should be rejected as it is over half of 14 Substitute an acceptable x into V ($0 < x < 7$) Volume = 576	B1 M1 A1 [3]	**ft** from *their* incorrect x Explanation necessary (e.g. one length is -ve) Alt: use of second derivative acceptable. Ignore units	"V is -ve" as the only explanation not accepted. Nor is "x is too big".

Question			Answer	Marks	Guidance	
12	**(i)**		Time out = $\frac{15}{x}$, Time back = $\frac{15}{x-2}$ Total time = $\frac{15}{x}+\frac{15}{x-2}$	**B1** **B1** **[2]**	For one Addition of two correct terms **isw**	
	(ii)		$\frac{15}{x}+\frac{15}{x-2}=6$ $\Rightarrow 15(x-2)+15x=6x(x-2)$ $15x-30+15x=6x^2-12x$ **oe** $\Rightarrow 6x^2-42x+30=0$ $\Rightarrow x^2-7x+5=0$	**B1** **M1** **A1** **A1** **[4]**	Equate *their* time to 6 Multiply throughout by LCM Brackets cleared **www** At least one interim step must be seen. N.B. Answer given	3 might be divided throughout here LCM implies 2 different algebraic denominators N.B. Algebra might have been done in (i)
	(iii)		$x^2-7x+5=0$ $\Rightarrow x=\frac{7\pm\sqrt{49-20}}{2}=\frac{7\pm\sqrt{29}}{2}$ **or** 6.19 and 0.807 (Paul's speed) = 6.19 km hr $^{-1}$ S.C (Paul's speed) is 6.19 km hr $^{-1}$ B2 or x = 6.19 B1	**M1** **A1** **A1** **[3]**	Use of correct formula with given equation + units but only if *their* 0.807 is discarded	Condone 0.81 or 0.8 but not 6.2 N.B. If 3 marks not awarded then look for the S.C.
	(iv)		$\frac{15}{x-2}-\frac{15}{x}=\frac{15}{4.19}-\frac{15}{6.19}$ Sight of 3.58 or 2.42 or 1.16 69 or 70 mins or 1hr 9 mins or 1 hr 10 mins	**M1** **B1** **A1** **[3]**	Sub *their* x into correct expression Or sub *their* x into the two separate correct expressions for time and then subtract Answer correct in minutes	Or the simplified expression Accept other way round giving a negative value

Question		Answer	Marks	Guidance	
13	(i)	$100x + 120y \leq 1200$ **oe** $2x + 1.5y \leq 18$ **oe**	**M1** **A1** **A1** **[3]**	Attempting to use information to create an inequality Ignore extra inequalities	Seen by one LH side Condone use of <
	(ii)		**B1** **B1** **B1** **B1** **[4]**	One line (allow intercepts ±0.1) Other line (allow intercepts ±0.1) Shading one line – 1st quad only Shading other line – 1st quad only N.B. Shading below a line gets B0	N.B. Candidates may get inequalities wrong in (i) but get the shading correct in (ii) - this should be allowed. Allow ft for shading of wrong line but only if the gradient is -ve Allow ft for shading of wrong line but only if gradient is -ve and the two lines intersect in the 1st quadrant (not on axes)
	(iii)	$(P =)\ 3.5x + 3y$ e.g. (9, 0) gives 31.5, (0, 10) gives 30 (4, 6) gives 32, (5, 5) gives 32.5 (3, 7) gives 31.5 (6, 4) gives 33 (6, 4) gives 33 **S.C**. for last 4 marks. (6,4) gives 33 B2 Either 33 or (6,4) B1	**B1** **M1** **A1** **A1** **A1** **[5]**	test at least two integer points in correct feasible region in correct OF Both points correct. Ignore any others For (6, 4) chosen For 33	Ignore any equating to a number i.e. the point must be identified as the maximum.

Question		Answer	Marks	Guidance	
14	(i)	Probability remains constant Imperfection of mugs independent	B1 B1 [2]		Not "Random"
	(ii)	$p=\frac{1}{20},\ q=\frac{19}{20}$	B1	Soi	
		$\text{P(0 or 1)} = \left(\frac{19}{20}\right)^{10} + 10\left(\frac{19}{20}\right)^{9}\left(\frac{1}{20}\right)$	M1 A1	$q^{10} + kpq^{9}$ attempted, k an integer > 0 $p + q = 1$ $k = 10$ **soi**	For k Not $\binom{10}{1}$ or ${}^{10}C_1$
		$= 0.5987 + 0.3151$ $= 0.9139$	A1	**soi** accept rounding to 3dp	
		$P(\geq 2) = 1 - 0.9139$ $= 0.086$	M1 A1 [6]	Dependent on previous M (Anything that rounds to 0.086)	
	(iii)	P(accepted) = P(0 imperfect) + P(1 imperfect) $\times$ P(0 imperfect)	M1	Correct plan **soi** by both correct terms	Words only sufficient
		$\left(\frac{19}{20}\right)^{10} + 10\left(\frac{19}{20}\right)^{9}\left(\frac{1}{20}\right)\left(\frac{19}{20}\right)^{10}$	A1 A1	Correct 1st term (as an expression) **soi** Correct 2nd term (as an expression including the 10) **soi**	**soi** by final answer
		$= 0.5987 + 0.1887$ $= 0.787(4...)$	A1 [4]	Accept 0.788	Anything that rounds to 0.787 or 0.788
		Alternative: P(accept) = 1 – (Ans to (ii) + P(1)×P(at least 1)) = 1 – (ans to (ii) + P(1)×(1–P(0))	M1	Correct plan soi by both correct terms taken from 1	Words only sufficient
		= 1 - (0.0861+ 0.3151 × 0.4013) = 1 - (0.0861+ 0.1264) =1 - 0.213	A1 A1	**Ft** Using *their* ans to (ii) 2nd term as an expression **soi** by final answer	
		= 0.787(4...)	A1		

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OCR
Oxford Cambridge and RSA

Wednesday 3 June 2015 – Morning

FSMQ ADVANCED LEVEL

6993/01 Additional Mathematics

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 6993/01

Other materials required:
- Scientific or graphical calculator

Duration: 2 hours

* 4 3 9 6 2 8 1 0 5 5 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

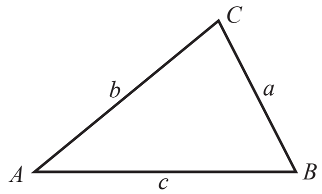
This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Formulae Sheet: 6993 Additional Mathematics



In any triangle *ABC*

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Binomial expansion

When n is a positive integer

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Section A

1 Find the equation of the line which is perpendicular to the line $2x+3y=5$ and which passes through the point (3, 4). **[3]**

2 **(i)** Find α in the range $0° \leqslant \alpha \leqslant 180°$ such that $\tan\alpha = -1.5$. **[2]**

(ii) Find β in the range $0° \leqslant \beta \leqslant 180°$ such that $\sin\beta = 0.2$. **[2]**

3 Find the equation of the tangent to the curve $y = x^3 + 3x - 5$ at the point (2, 9). **[5]**

4 **(i)** Find $\int_1^2 (x^2 + 2x + 3)\,\mathrm{d}x$. **[4]**

(ii) Interpret your answer geometrically. **[1]**

5 A train accelerates from rest from a point O such that at t seconds the displacement, s metres from O, is given by the formula $s = \frac{3}{2}t^2 - 2t + 3$.

(i) Show by calculus that the acceleration is constant. **[3]**

(ii) Find the velocity after 5 seconds. **[2]**

6 You are given that n is a positive integer and $(n-1)$, n, $(n+1)$ are three consecutive integers.

In each of the following cases form an equation in n and solve it.

(i) The three integers add up to 99. **[2]**

(ii) When the product of the first integer and third integer is added to 5 times the second integer the sum is 203. **[4]**

7 **(i)** Solve algebraically the simultaneous equations $y = 3 + 5x - x^2$ and $y = x + 7$. **[4]**

(ii) Interpret your answer geometrically. **[1]**

8 The cubic polynomial $\mathrm{f}(x) = x^3 + ax + 6$, where a is a constant, has a factor of $(x + 3)$.

(i) Find the value of a. **[2]**

(ii) Hence or otherwise, solve the equation $\mathrm{f}(x) = 0$ for this value of a. **[4]**

9 The equation of the circle C is $x^2 + y^2 - 8x + 2y - 19 = 0$.

(i) Express the equation of C in the form $(x - a)^2 + (y - b)^2 = r^2$. **[4]**

(ii) Hence or otherwise, use an algebraic method to decide whether the point $(8, 3)$ lies inside, outside or on the circumference of the circle.
Show all your working. **[2]**

10 Fig. 10 shows a partly open window OA, viewed from above. The window is hinged at O. When the window is closed, the end A is at point B. The window is kept open by a rod CD, where C is a fixed point on the line OB.
The point D slides along a fixed bar EF. When the window is closed, D is at F. When the window is fully open, D is at E.

$\mathrm{OA} = \mathrm{OB} = 20\,\mathrm{cm}$, $\mathrm{OC} = 8\,\mathrm{cm}$, $\mathrm{CD} = 7\,\mathrm{cm}$, $\mathrm{EF} = 5\,\mathrm{cm}$, $\mathrm{OE} = 10\,\mathrm{cm}$

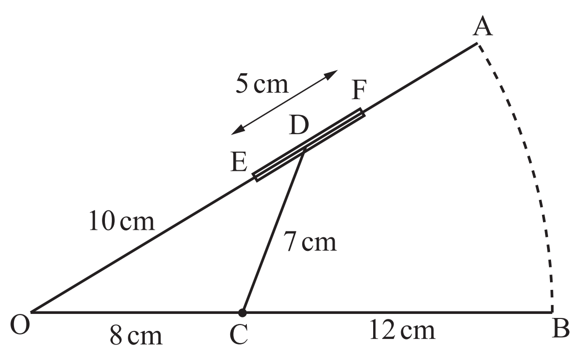


Fig. 10

Find

(i) angle EOC when the window is fully open, **[3]**

(ii) the distance OD when angle EOC is 30°. **[4]**

Section B

11 Two curves, S_1 and S_2 have equations $y = x^2 - 4x + 7$ and $y = 6x - x^2 - 1$ respectively. The curves meet at A and at B.

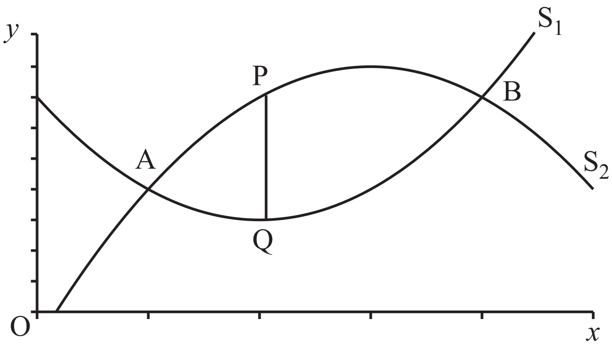


Fig. 11

(i) Show that the coordinates of A and B are (1, 4) and (4, 7) respectively. **[2]**

Points P and Q lie on S_2 and S_1 between A and B. P and Q have the same x coordinate so that PQ is parallel to the y-axis, as shown in Fig. 11.

(ii) Find an expression, in its simplest form, for the length PQ as a function of x. **[2]**

(iii) Use calculus to find the greatest length of PQ. **[4]**

(iv) Find the area between the two curves. **[4]**

12 A distributor of flower bulbs has a large number of tulip bulbs and daffodil bulbs, mixed in the ratio 1 : 3 respectively. He packs the bulbs in boxes. He puts 10 bulbs, chosen at random, into each box.

(a) Find the probability that a box, chosen at random, contains

(i) exactly 4 daffodil bulbs, **[4]**

(ii) at least 1 tulip bulb. **[3]**

(b) Two boxes of bulbs are chosen at random.

Find the probability that there is a total of 3 tulip bulbs in the two boxes. **[5]**

13 A gardener marks out a regular hexagon ABCDEF on his horizontal garden.
Each side of the hexagon is 0.5 m. The gardener sticks a cane in the ground at each point of the hexagon. He joins the six canes at V where V is vertically above the centre, O, of the hexagon, as shown in Fig. 13. Each cane has a length of 2.4 m from the ground to V.

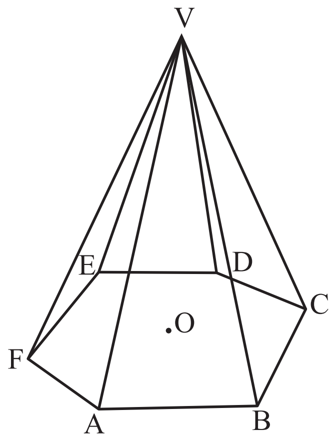


Fig. 13

Calculate, giving your answers to 3 significant figures,

(i) the vertical height of V above the ground, **[3]**

(ii) the angle between each cane and the ground, **[2]**

(iii) the angle between the plane VAB and the ground. **[4]**

The gardener stretches a horizontal wire around the structure to strengthen it. He fixes the wire to each cane at a point 1 m vertically above the ground.

(iv) Find the length of the wire. **[3]**

14 A company produces bottles of two liquids, X and Y. There are two ingredients, A and B, in each liquid.

The table shows the quantities, in centilitres (cl), of A and B needed for each bottle of liquid.

	A	B
X	4	2
Y	3	5

Each day the company can use 84 cl of A and 90 cl of B.

From this information an analyst writes down the inequality $4x+3y \leqslant 84$.

(i) Explain what x and y stand for in this inequality and explain what the inequality models. **[2]**

(ii) Use the information given to write down another inequality, other than $x \geqslant 0$ and $y \geqslant 0$. **[1]**

(iii) On the grid given in the answer booklet, illustrate your two inequalities. Shade the region that is not required. **[3]**

(iv) The company needs to produce the same number of bottles of X and of Y each day.

Find the maximum number of bottles of X and of Y that the company can produce. **[2]**

(v) On one day the company does not have to produce the same numbers of bottles of X and of Y.

Write down the maximum number of bottles that can be produced and all the combinations that will give this maximum. **[4]**

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OCR
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FSMQ

Additional Mathematics

Unit **6993:** Additional Mathematics

Free Standing Mathematics Qualification

Mark Scheme for June 2015

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
	Method mark dependent on a previous mark, indicated by "Dep on 1st M"
cao	Correct answer only
oe	Or equivalent
soi	Seen or implied
www	Without wrong working

1. **Marking Instructions**

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Section A

Question			Answer	Marks	Guidance	
1			Line is $3x-2y=k$ Satisfied by (3, 4) ($\Rightarrow k = 1$) $\Rightarrow 3x - 2y = 1$	**B1** **M1** **A1** **3**	oe Substitution soi oe isw Only 3 terms	
			Alternatively: $g=-\frac{2}{3} \Rightarrow$ new $g=\frac{3}{2}$ $\Rightarrow (y-4)=\frac{3}{2}(x-3)$ $\Rightarrow 2y=3x-1$	**B1** **M1** **A1**	Soi from equation *Their* normal gradient and (3,4) used oe isw Only 3 terms	Condone $g=\frac{3}{2}x$ Only allow if correct or if their g and the negative reciprocal is seen. i.e. only one constant term
			Alternatively: $g=-\frac{2}{3} \Rightarrow$ new $g=\frac{3}{2}$ $\Rightarrow y=\frac{3}{2}x+c$ $\Rightarrow y=\frac{3}{2}x-\frac{1}{2}$	**B1** **M1** **A1**	Soi from equation *Their* normal gradient and (3,4) substituted oe isw Only 3 terms	Condone $g=\frac{3}{2}x$ Only allow if correct or if their g and the negative reciprocal is seen. e.g. $y=\frac{3x-1}{2}$

Question			Answer	Marks	Guidance	
2	(i)		$\tan\alpha = -1.5 \Rightarrow \alpha = 123.69(.......)°$ or 123.7° or 124°	B2 2	**B1** sight of ±56.3° or 304° or 303.7° or 303.69°.... or 123.6° −1 from full marks for extra values in range. Ignore values outside range.	Ignore lack of degree sign. No marks for answers in radians.
	(ii)		11.5° or anything that rounds to 11.5° and 168° or anything that rounds to168.5°	B1 B1 2	−1 from full marks for extra values in range. Ignore values outside range.	Ignore lack of degree sign. No marks for answers in radians.

Question			Answer	Marks	Guidance	
3			$\frac{dy}{dx} = 3x^2 + 3$ $\Rightarrow g = 15$ $\Rightarrow (y-9) = 15(x-2)$ $\Rightarrow y = 15x - 21$	M1 A1 A1 M1 A1 5	Diffn Both terms isw Gradient cao Correct form for line with *their* gradient and (2,9) used. Dep on 1st M mark Three terms only	Powers reduced by 1 in at least one x term. M0 for $x^2 + 3$ Ignore any $+c$ Alt: Use $y = mx + c$ with *their* g and substitute (2,9) NB No calculus, no marks

Question			Answer	Marks	Guidance	
4	(i)		$\int_1^2 (x^2+2x+3)\,dx = \left[\frac{x^3}{3}+x^2+3x\right]_1^2$ $=\left(\frac{8}{3}+4+6\right)-\left(\frac{1}{3}+1+3\right)$ $=12\frac{2}{3}-4\frac{1}{3}=8\frac{1}{3}$ **oe**	**M1** **A1** **M1** **A1** **4**	Int All three terms (ignore c) Apply limits and subtract in correct order. soi Dep on 1st M www	Powers increased by 1 in at least one term M0 for x^3+2x^2+3x Condone lack of brackets. $-8\frac{1}{3}$ is by implication M0.
	(ii)		Area between $y=x^2+2x+3$, $x = 1$, $x = 2$ and the x axis. (Or "under" or "below" with no mention of x axis)	**B1** **1**	Allow sketch with area shaded and $x = 1$ and $x = 2$ clearly seen. Curve in 1st quadrant and right way up.	May write "curve" (but not "line") instead of equation. Do not allow anything that refers to points (1,0), (2,0) and not lines.

Question			Answer	Marks	Guidance	
5	(i)		$s=\frac{3}{2}t^2-2t+3 \Rightarrow \left(v=\frac{ds}{dt}=\right)3t-2$ $\Rightarrow a$ or $\frac{dv}{dt}$ or $\frac{d^2s}{dt^2}=3$	**M1** **B1** **A1** **3**	Diffn twice For sight of $3t-2$ www correctly defined or in words	Powers decreased by 1 in at least one term. M0 for $\frac{3}{2}t-2$
	(ii)		$v = 3 \times 5 - 2 = 13$ Velocity = 13 (m s^{-1})	**M1** **A1** **2**	Substitute into *their* v Ignore units	Note that use of SUVAT formulae will give different answers. Use of $v = u + at$ to give 15 M1 A1 You may also see 12.2, 15, 13.6, $\sqrt{183}\approx 13.5(....)$

Question			Answer	Marks	Guidance	
6	(i)		$(n-1)+n+(n+1)=99$ or $3n=99$ $\Rightarrow n=33$	B1 B1 2	Must be seen isw Isw 32,33,34 unidentified is B0 (n) = 33 without the eqn is B1	
	(ii)		$(n-1)(n+1)+5n=203$ $\Rightarrow n^2+5n-204(=0)$ $\Rightarrow (n+17)(n-12)=0$ $\Rightarrow n=12$	B1 B1 M1 A1 4	Correct 3 term quadratic either as expression or equation implies first B1 To give $(n\pm12)(n\pm17)$ soi n = –17 must be rejected (by ignoring it is acceptable) isw	e.g. $n^2+5n=204$ Or correct sub from correct quadratic in correct formula Or sight of 6.25 (soi) in completing the square to give $(n\pm2.5)^2=204+6.25$ If no equation seen then n = 12 is M1 A1, n =12 and –17 is M1 A0 For last two marks: If trial on correct equation gives n = 12 then M1 A1

Question			Answer	Marks	Guidance	
7	(i)		$\Rightarrow x+7=3+5x-x^2$ $\Rightarrow x^2-4x+4=0$ oe $\Rightarrow x=2,$ $y=9$	**M1** **A1** **A1** **A1** **4**	Substitute, eliminating x or y. 3 term quadratic. x(or y) Substitute and find y (or x).	Or $y^2-18y+81=0$
	(ii)		Line is tangent to curve (at (2, 9))	**B1** **1**	Allow "touches".	Or a sketch with any parabola touched by any line

Question			Answer	Marks	Guidance	
8	(i)		$f(-3)=(-3)^3-3a+6=0$ $\Rightarrow 3a=-21$ $\Rightarrow a=-7$	**M1** **A1** **2**	Sub –3 and equating to 0. Allow 27 – 3a + 6 = 0	Or equivalent by long division and equating remainder to 0. Or Use $(x+3)(x^2+px+q)=f(x)$ and equate coefficients
	(ii)		x^2-3x+2 or x^2+x-6 or x^2+2x-3 $(x+3)(x-2)(x-1)$ $x=1,\ 2,\ -3$	**B1** **B1** **B1** **B1** **4**	Quadratic $(x\pm 2)$ $(x\pm 1)$ Ans only B4 NB. Quadratic can be recovered even if cubic is wrong.	Alternative method by factor theorem Any trial except f(–3) B1 Obtain f(1) = 0 B1 Obtain f(2) = 0 B1 Ans B1

Question			Answer	Marks	Guidance	
9	(i)		$(x-4)^2$	B1	Sight of $(x \pm 4)^2$	
			$(y+1)^2$	B1	Sight of $(y \pm 1)^2$	
			36	B1	Sight of –36 on lhs or 36 on rhs	
			$\Rightarrow (x-4)^2 + (y+1)^2 = 6^2$	B1	Allow 36. isw	
				4		
	(ii)		Distance from *their* centre to (8, 3) $= \sqrt{(8-4)^2 + (3+1)^2} = \sqrt{16+16} = \sqrt{32}$	M1	By pythagoras	Or sub into either form of the circle.
			and $\sqrt{32} < 6$ or $32 < 36$ so inside	A1	Reason must be given	< 0 (or < 36) therefore inside
			NB. Their centre is either stated or assumed to be their (4, –1) in (i)			
				2		

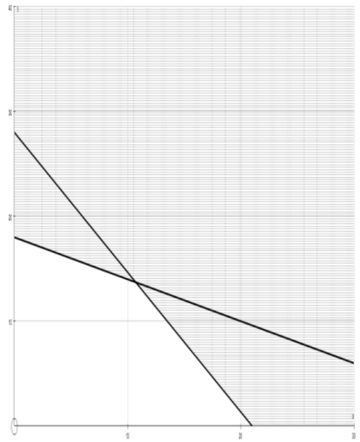
Question			Answer	Marks	Guidance	
10	(i)		$\cos\theta = \dfrac{10^2+8^2-7^2}{2\times8\times10}$	M1	Correct substitution into correct formula to give correct angle.	Could be done in 2 stages
			$= 0.71875$	A1	Accept 3sf Soi by answer	
			$\Rightarrow \theta = 44^{\circ}$ or $44.0....^{\circ}$ or 44.1°	A1 3		
	(ii)		$7^2 = x^2 + 8^2 - 2\times x\times 8\times\cos 30$	M1	Correct substitution into correct formula	
			$= x^2 + 64 - 13.86x$			
			$\Rightarrow x^2 - 13.86x + 15 = 0$	A1 M1	Oe. Accept 13.9 or better Solve *their* quadratic with correct substitutions into correct formula. Dep on 1st M1	
			x = 12.7 cm	A1 4		
			Alternatively: Sin rule to find one angle	M1	Correct substitution into correct formula	
			D = 34.8(5)°, C = 115.1(5)°	A1		
			Find other angle and use sin rule or cosine rule	M1 A1	Correct substitution into correct formula	

Section B

Question			Answer	Marks	Guidance	
11	(i)		Substitute A(1, 4) into both equations Substitute B(4, 7) into both equations	B1 B1 2	NB. Answer given so working must be seen	Alternatively by solving: Forming the correct three term quadratic by equating B1 e.g. $2x^2-10x=-8$ Both pairs of coordinates found B1
	(ii)		$\text{PQ}=\left(6x-x^2-1\right)-\left(x^2-4x+7\right)$ $=10x-2x^2-8$	M1 A1 2	Allow QP. Allow no brackets Cao isw	
	(iii)		$\frac{\text{dPQ}}{\text{d}x}=10-4x$ $=0$ when $x=2.5$ $\Rightarrow \text{PQ}=4.5$	M1 A1 M1 A1 4	In *their* quadratic PQ at least one power decreased by 1. Allow x = 2.5 from diffn of QP and PQ/2 and QP/2 Substitute *their* x into *their* quadratic. Dep on first M mark Cao www (NB only from 10 – 4x)	NB Non calculus methods get M0 Functions can be diffn separately and equated for marks Look out for the abandonment of the –ve sign from use of QP
	(iv)		$A=\int_1^4\left(10x-2x^2-8\right)\text{d}x$ $=\left[5x^2-2\frac{x^3}{3}-8x\right]_1^4$ $=5\frac{1}{3}-\left(-3\frac{2}{3}\right)=9$	M1 A1 M1 A1 4	Int of *their* PQ; at least one power increased by 1. Correct integrand. From correct PQ only. Ignore c and limits. Apply limits and subtract in correct order. soi Dep on 1st M (need not be correct integrand) www	Alternatively: Finding the area under each curve Integrate each function M1 Correct integrands A1 Apply limits and subtract in correct order and then subtract answers in correct order M1 Ans 21 – 12 = 9 A1 www

Question			Answer	Marks	Guidance	
12	(a)	(i)	$\binom{10}{4}\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^6 = 210\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^6$ $= 0.0162$	M1 A1 A1 A1 4	$0.75^n \times 0.25^{10-n}$ seen, $n \neq 0$ or 10 Coefficient evaluated soi by answer Powers soi by answer SC. Use of $^1/_3$, $^2/_3$ in correct order gives 0.0569 B2	Answers throughout to 3sf or better. Allow % probabilities NB 0.75 and 0.25 interchanged is max 2/4 (gives ans = 0.146)
	(a)	(ii)	$\text{P(0 tulip bulbs)} = \left(\frac{3}{4}\right)^{10}$ $(= 0.0563)$ $\text{P(at least 1 tulip bulb)} = 1 - \left(\frac{3}{4}\right)^{10} = 1 - 0.0563$ $= 0.944$	B1 M1 A1 3	For $\left(\frac{3}{4}\right)^{10}$ seen (possibly amongst other terms) For $1 - p^{10}$ used. p can also be $^1/_4$, $^2/_3$	Alternatively: Add 10 terms M1 Correct powers and coefficients soi A1 Ans A1
	(b)		$\text{P(3 tulips in 20)} = \binom{20}{3}\left(\frac{3}{4}\right)^{17}\left(\frac{1}{4}\right)^3 = 1140\left(\frac{3}{4}\right)^{17}\left(\frac{1}{4}\right)^3$ $= 0.134$	B1 M1 A1 A1 A1 5	Sight of 20 anywhere.soi $0.75^n \times 0.25^{20-n}$ seen, $n \neq 0$ or 20 Coefficient evaluated soi by answer Powers soi by answer	
			Alternatively: 4 terms or 2 terms 0 in one and 3 in the other + 1 in one and 2 in the other 0.0141 (× 2 = 0.0282) 0.0529 (× 2 = 0.1057) 0.134	B1 M1 A1 A1 A1	Sight of 0,3 and1,2 One of these pairs multiplied, each term of the form $0.75^n \times 0.25^{10-n}$ rounds to 0.014 soi rounds to 0.053 soi	So e.g. 0.067 from 2 terms is 4/5

Question			Answer	Marks	Guidance	
13	(i)		OA = 0.5 Pythagoras $\left(OV^2\right) = 2.4^2 - \textit{their}\ OA^2$ $\Rightarrow OV = 2.35\text{m}$	B1 M1 A1 3	Or any line from centre to corner of base Do not accept $\sqrt{5.51}$ for final answer. Accept better than 3sf	
	(ii)		$\cos^{-1}\dfrac{\textit{their}\ OA}{2.4}$ $= 78.(0)°$	M1 A1 2	OR $\sin^{-1}\dfrac{\textit{their}\ OV}{2.4} = 78.0°$ OR $\tan^{-1}\dfrac{\textit{their}\ OV}{\textit{their}\ OA} = 78.0°$ Accept 77.9° – 78.3°	
	(iii)		Angle is VMO where M is the midpoint of AB. $OM^2 = \textit{their}\ OA^2 - 0.25^2$ $\Rightarrow OM = 0.433$ or $VM = 2.39$ oe Angle VMO = $\tan^{-1}\dfrac{\textit{their}\ OV}{\textit{their}\ OM} = 79.5°$	 M1 A1 M1 A1 4	 OM or VM Using triangle VMO Accept 79.5° – 79.6°	
	(iv)		$\dfrac{1}{\textit{their}\ OV}$ up $\dfrac{\textit{their}\ OV - 1}{\textit{their}\ OV} = \dfrac{1.35}{2.35}\left(= \dfrac{27}{47} = 0.575\right)$ $\times 3 = 1.72$	M1 A1 A1 3	Attempt by correct ratio Ratio using slant edges $\dfrac{2.04 - 1.02}{2.4}$	NB. 1.4/2.4 is incorrect and gives 1.75 so M0 Alternatively: Use of trigonometry in truncated hexagon (ht 1.35) M1 gives length of base 0.287 A1 × 6 = 1.72 A1 Beware lots of alternatives!

Question			Answer	Marks	Guidance	
14	(i)		x is the number of bottles of X (produced), y is the number of bottles of Y (produced). (The constraint) models the quantity of A	B1 B1 2	Both	
	(ii)		$2x + 5y \leq 90$	B1 1	Do not accept <	
	(iii)			B1 B1 B1 3	One line (0, 28) to (21,0) Other line (0,18) to (30,6) Shading ft for two lines with negative gradients and which intersect in 1st quadrant.	Allow up to one small square out at each edge of grid.
	(iv)		(12, 12) or 12 (of each)	B2 2	Give B1 for attempt to find by drawing line $y = x$ or testing (n,n) or an answer of 24.	Line $y = x$ can be seen on previous graph
	(v)		Maximum is 24 (10, 14), (11, 13), (12, 12)	B1 B3 4	B1 for each. If all 3 given then – 1 for each extra one.(Ignore same point given twice)	Condone for e.g. 10X + 14Y

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OCR
Oxford Cambridge and RSA

Monday 6 June 2016 – Afternoon

FSMQ ADVANCED LEVEL

6993/01 Additional Mathematics

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 6993/01

Other materials required:
- Scientific or graphical calculator

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

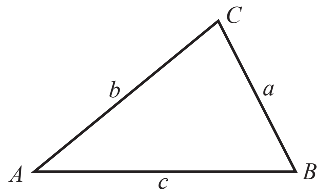
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

Formulae Sheet: 6993 Additional Mathematics

In any triangle ***ABC***

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$



Binomial expansion

When n is a positive integer

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Section A

Answer **all** the questions.

1 Solve the inequality $1-2(x-3) > 4x$. **[3]**

2 The gradient function of a curve is given by $\frac{dy}{dx} = 3x^2 - 4x + 2$.

Find the equation of the curve, given that it passes through the point (1, 3). **[4]**

3 Find all the values of x in the range $0° < x < 360°$ that satisfy $3\sin x = 4\cos x$. **[4]**

4 You are given that $f(x) = x^3 - x^2 + x - 6$.

Show that

(i) $(x - 2)$ is a factor of $f(x)$, **[1]**

(ii) the equation $f(x) = 0$ has only one real root. **[4]**

5 John draws a triangle ABC with sides AB = 12 cm, BC = 16 cm and AC = 20 cm. However, he can only measure the sides to the nearest centimetre.

(i) State the smallest possible length of AB in John's drawing. **[1]**

(ii) Hence calculate the largest possible value of the angle B in John's drawing. **[3]**

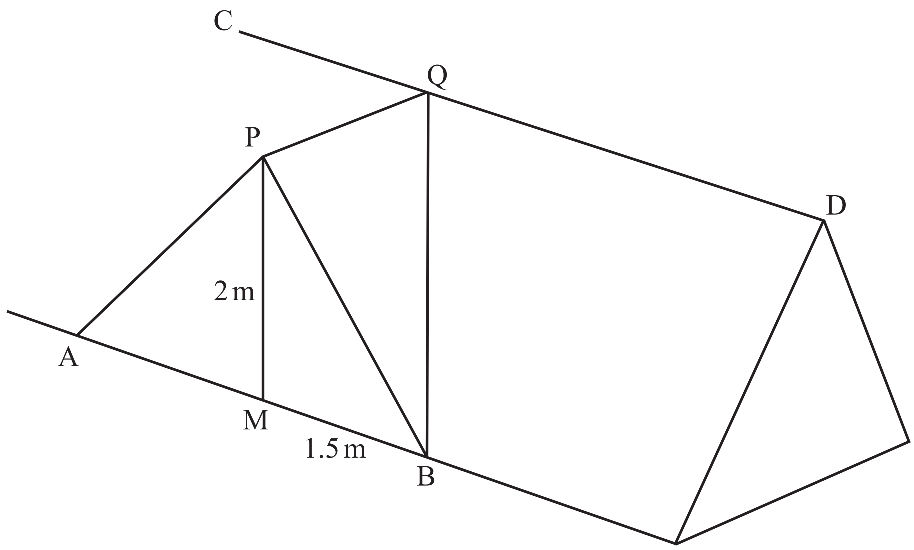
6 Two cars are initially at rest facing in the same direction on a straight road. Car A is 100 m ahead of car B. The two cars start from rest at the same moment.
Car A moves with constant acceleration of $1.5\,\mathrm{m\,s^{-2}}$ and car B moves with constant acceleration of $2\,\mathrm{m\,s^{-2}}$.

Find

(i) the distance that car B travels before it overtakes car A, **[4]**

(ii) the speed of car B at the moment when it overtakes car A. **[2]**

7 An extension to the roof of a house is shown in the diagram below.



The ridge, CD, and the lines AB and PQ are horizontal. PQ is perpendicular to CD.
M is the midpoint of AB. The line PM is vertical.

APB is an isosceles triangle with height 2 metres and base length 3 metres.
Angle PQM is 45°.

Find

(i) the length of PQ, **[1]**

(ii) the angle PBQ. **[4]**

8 **(i)** Write down the binomial expansion of $(1 + \delta)^3$. **[2]**

(ii) Hence explain why, if δ is small, $(1 + \delta)^3 \approx 1 + 3\delta$.
[$\approx$ means 'is approximately equal to'] **[1]**

You are given that the equation $x^3 - 0.9x - 0.206 = 0$ has a root very close to $x = 1$.

(iii) Substitute $x = 1 + \delta$ into the equation and use the approximation in part **(ii)** to find an estimate of this root, correct to 3 significant figures. Show all your working. **[4]**

9 A curve has equation $y = x^3 - 3x^2 - 3x + 4$.
Points P and Q lie on the curve. The coordinates of P are $(3, -5)$.

(i) Find the equation of the tangent to the curve at P. **[4]**

The tangent to the curve at Q is parallel to the tangent to the curve at P.

(ii) Find the coordinates of Q. **[3]**

10 **(i)** On the axes given in the Printed Answer Book, indicate the region for which the following inequalities hold. You should shade the region that is **not** satisfied by the inequalities.

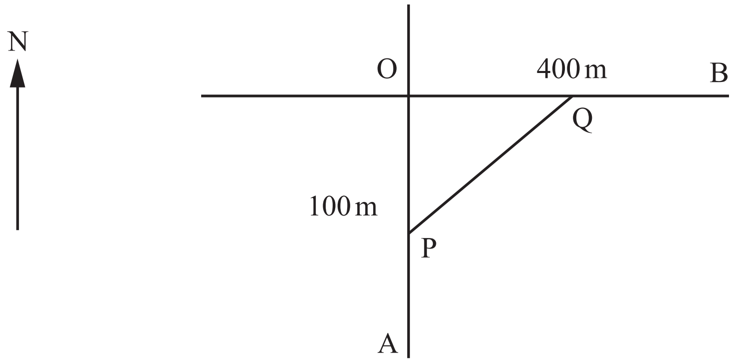
$$4x + 3y \leqslant 30$$
$$y \geqslant 2x$$
$$x \geqslant 1$$

[5]

(ii) Find the maximum value of $7x + 4y$ subject to these conditions. **[2]**

Section B

11 A railway track runs due east-west and is crossed at O by a road running due south-north, as shown below. The crossing has no barriers.



Initially a train is at point B, 400 m from O, and a car is at point A, 100 m from O. The train is travelling at a constant speed of $25\,\text{m}\,\text{s}^{-1}$ towards O and the car is travelling at a constant speed of $20\,\text{m}\,\text{s}^{-1}$ towards O.

At time t seconds the train is at point Q and the car is at point P.

(i) Find expressions for the distances OP and OQ as functions of t. **[2]**

(ii) The distance between the car and the train at time t s is x m. Find a formula for x^2 in terms of t. Give your formula in the form $x^2 = a + bt + ct^2$ where a, b and c are to be determined. **[3]**

(iii) Differentiate this formula with respect to t and find the time at which x^2 is a minimum. Hence find the shortest distance between the car and the train. **[6]**

(iv) Show that the car passes point O before the train. **[1]**

12 The line L_1 has equation $3x - y = 1$ and the point P has coordinates (8, 3).

(i) Find the equation of the line L_2 which passes through P and is perpendicular to line L_1. **[3]**

(ii) Find the coordinates of the point Q where L_1 and L_2 intersect. **[3]**

(iii) Find the length PQ. **[2]**

(iv) Write down the equation of the circle that has centre P and line L_1 as a tangent. **[1]**

(v) Find the equation of the other line that is a tangent to the circle and is parallel to line L_1. **[3]**

13 The cost of a packet of buns in a local supermarket is x pence and the cost of a loaf of bread is $x + 75$ pence.

(i) Write an expression for the number of packets of buns that can be bought for £5.40 and an expression for the number of loaves that can be bought for £5.40. **[2]**

The number of packets of buns that can be bought for £5.40 is 5 more than the number of loaves that can be bought for £5.40.

(ii) Using this information and your answer to part **(i)**, derive an equation in x and show that it simplifies to $x^2 + 75x - 8100 = 0$. **[5]**

(iii) Solve this equation to find the cost of a packet of buns and the cost of a loaf of bread. **[5]**

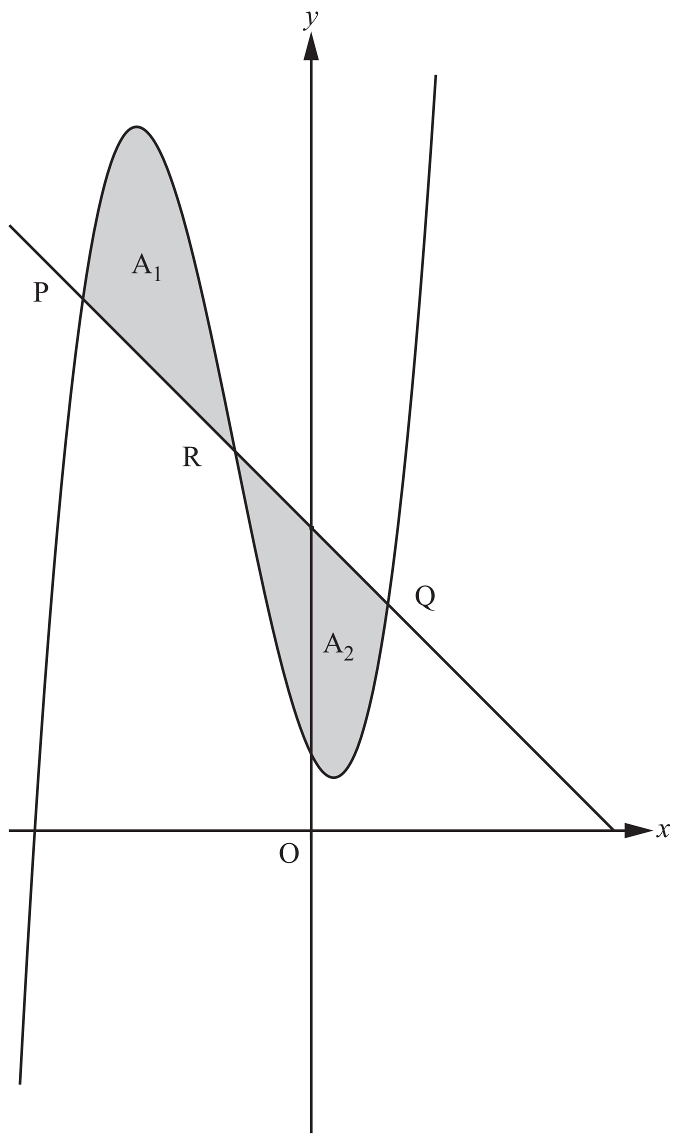
Question 14 is printed overleaf

14 The equation of a curve is given by $y = x^3 + ax^2 + bx + 1$. The points P $(-3, 7)$ and Q $(1, 3)$ lie on the curve.

(i) Form two equations in a and b. Solve these equations to show that $a = 3$ and $b = -2$. **[4]**

(ii) Find the midpoint, R, of the line PQ and show that R lies on the curve. **[2]**

The diagram below shows the curve and the line PRQ.



The area between the curve and the line segment PR is A_1 and the area between the curve and the line segment RQ is A_2.

(iii) Show that $A_1 = A_2$. **[6]**

END OF QUESTION PAPER

Oxford Cambridge and RSA

FSMQ

Additional Mathematics

Unit **6993:** Additional Mathematics

Free Standing Mathematics Qualification

Mark Scheme for June 2016

Annotations and abbreviations

Annotation	Meaning
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
Isw	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
AG	Answer given
M1 dep	Method mark dependent on a previous method mark(s)
cao	Correct answer only
oe	Or equivalent
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for Additional Mathematics

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 can never be awarded.

B
Mark for a correct result or statement independent of Method marks.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

Section A

Question			Answer	Marks	Guidance	
1			$1-2(x-3)>4x$ $\Rightarrow a>bx$ or $-a<-bx$ Either $a=7$ or $b=6$ in either of above $\Rightarrow x<\frac{7}{6}$ $\left(\text{or } 1.17 \text{ or } 1.1\dot{6}\right)$	**M1** **A1** **A1**	Expand and collect soi www isw	Do not allow = anywhere even if final answer correct
				3		

Question			Answer	Marks	Guidance	
2			$\frac{dy}{dx}=3x^2-4x+2$ $\Rightarrow y=x^3-2x^2+2x\ (+c)$ **oe** Satisfied by (1, 3) $\Rightarrow 3=1-2+2+c$ $(\Rightarrow c=2)$ $\Rightarrow y=x^3-2x^2+2x+2$	**M1** **A1** **M1dep** **A1**	Int: At least 1 power increased by 1: Beware mult by x Three terms ignoring c Substitution Complete simplified equation	
				4		

Question			Answer	Marks	Guidance	
3			$3\sin x = 4\cos x \Rightarrow \tan x = \frac{4}{3}$ $\Rightarrow x = 53.1(3)$ and $x = 180 + 53.13 = 233(.13)$ Alternative: Square, use Pythagoras M1 $\Rightarrow \cos x = \pm 0.6$ or $\sin x = \pm 0.8$ A1(must include ±) Gives 53.1 A1 Or 233 B1 only if no extra values in range	**M1** **A1** **A1** **B1**	For tan*x* For $\frac{4}{3}$ One angle (53 not acceptable) ft Other angle B0 any extra values in range, ignore any outside range	Allow $\tan^{-1}\left(\frac{4}{3}\right) = k$ or $\tan^{-1}\left(\frac{3}{4}\right) = k$
				4		

Question			Answer	Marks	Guidance	
4	(i)		$8 - 4 + 2 - 6 = 0$ Alternative: Demonstration that $f(x) = (x-2)(x^2 + x + 3)$	**B1**	must be seen	i.e. powers evaluated
				1		
	(ii)		$f(x) = (x-2)(x^2 + x + 3)$ $D = b^2 - 4ac = 1 - 4 \times 3\ (= -11)\ (< 0)$ or $(x + 0.5)^2 + 2.75 \neq 0$ so only one root or no other roots	**M1** **A1** **M1** **A1**	Factorise: Any 2 correct terms of 3 term quadratic seen. For long division: first two terms For quad factor Numerical evidence must be seen on correct quadratic. Last statement must be seen. Condone reference to (*x* – 2) being the root.	If quad factor is found in (i) then give credit in (ii) if seen in (ii) e.g. $\sqrt{-11}$ won't work
				4		

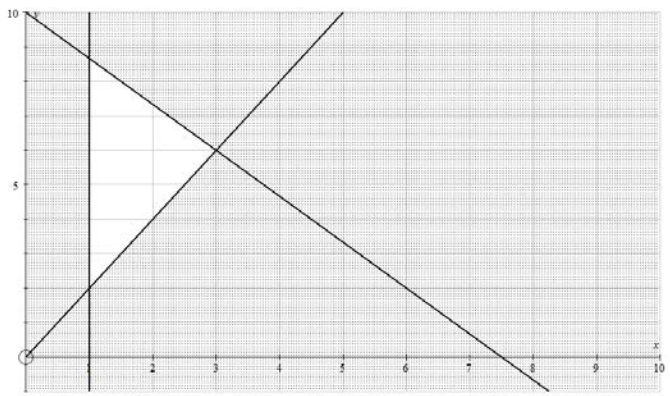
Question			Answer	Marks	Guidance	
5	(i)		11.5	B1	One number only seen or AB clearly identified	
				1		
	(ii)		Use 11.5, 15.5 and 20.5 $\cos B = \frac{11.5^2 + 15.5^2 - 20.5^2}{2 \times 11.5 \times 15.5}$ $(= -0.1339...)$ $\Rightarrow B = 97.7^0$	B1 M1 A1	Correct use of cosine rule on correct angle using values rounding to given values Answers rounding to 97.7	i.e. range [11.5,12.5]. [15.5,16.5],[19.5,20.5] Values must be consistent.
				3		

Question			Answer	Marks	Guidance	
6	(i)		$\left(\text{Distance for A:}\right) \frac{3}{4}t^2$ $\left(\text{Distance for B: }\right) t^2$ $\Rightarrow s = \frac{3}{4}(s \pm 100)$ or $s \pm 100 = \frac{3}{4}s$ or $\frac{3}{4}t^2 \pm 100 = t^2$ $\Rightarrow s = 300$ or 400 or $t = 20$ $\Rightarrow$ B travels 400 m	B1 B1 M1 A1	soi; ignore 100 soi; ignore 100 Equating distances leading to one of the 6 forms www	SC4 www for trial and error giving correct answer.
				4		
	(ii)		Using $v^2 = u^2 + 2as$ $\Rightarrow v^2 = 2.2.400 = 1600$ $\Rightarrow v = 40 \text{ m s}^{-1}$	M1 A1	And using a = 2 and *their* s from (i) www	Or complete and equivalent method. Allow missing u
				2		

Question			Answer	Marks	Guidance	
7	(i)		2	B1		
				1		
	(ii)		For PB: $PB = \sqrt{2^2 + 1.5^2} = 2.5$ $\Rightarrow \tan PBQ = \frac{2}{2.5} = 0.8$ $\Rightarrow \text{Angle PBQ} = 38.7^\circ$	M1 A1 M1 A1	Using *their* PQ and PB Alternatively for the last two marks: Attempt to find QB and use it with sin, cos or sine rule or cosine rule	If PB is found in (i) then give credit in (ii) only if seen in (ii). n.b. $QB = \sqrt{10.25}$
				4		

Question			Answer	Marks	Guidance	
8	(i)		$(1+\delta)^3 = 1^3 + 3.1^2\delta + 3.1\delta^2 + \delta^3$ $= 1 + 3\delta + 3\delta^2 + \delta^3$	B1 B1	Unsimplified expansion soi	Can be by expansion
				2		
	(ii)		Because, if δ is small, then (terms in) $(3)\delta^2$ and δ^3 are very small and can be ignored	B1	"ignored" or similar must be seen	e.g. neglected, eliminated
				1		
	(iii)		$(1+\delta)^3 - 0.9(1+\delta) - 0.206 \quad (=0)$ $\Rightarrow 1 + 3\delta - 0.9(1+\delta) - 0.206 \quad (=0)$ $\Rightarrow 2.1\delta = 0.106$ $\Rightarrow \delta = 0.05.....$ $\Rightarrow x = 1.05...$	M1 M1dep A1 A1	Sub Using result of (ii) 3sf or better	
				4		

Question			Answer	Marks	Guidance	
9	(i)		$\frac{dy}{dx} = 3x^2 - 6x - 3$	M1	Diffn. At least one power reduced by 1. Beware division by x	Ignore $+c$
			When $x = 3, \frac{dy}{dx} = 6$	A1		
			$\Rightarrow$ Equation of tangent is $y + 5 = 6(x - 3)$ **oe**	M1dep	Any valid form using *their* gradient and (3, –5).	
			$\Rightarrow y = 6x - 23$ **oe**	A1	**oe** only 3 terms	
				4		
	(ii)		$\frac{dy}{dx} = 3x^2 - 6x - 3 = 6$	M1	Equating *their* gradient fn and *their* 6	
			$\Rightarrow x^2 - 2x - 3 = 0$			
			$\Rightarrow (x-3)(x+1) = 0$	A1	Correct factorisation www	
			$\Rightarrow$ Q is where $x = -1, y = 3$	A1	cao www	Ignore (3, –5) as a possible answer. SC3 if $\frac{dy}{dx} = x^2 - 2x - 1 \Rightarrow$ g = 2 in (i) and Q is correct.
				3		

Question			Answer	Marks	Guidance	
10	**(i)**			**B1** **B1** **B1** **B1** **B1**	One line 2nd line 3rd line Shading *x* ≤ 1 Other shading. Allow ft if gradients of lines are the same sign as the correct lines.	Allow one small square tolerance on each axis
				5		
	(ii)		Max value at intersection which is (3, 6) =45	**B1** **B1**	soi	e.g. 45 gets 2
				2		

Section B

Question			Answer	Marks	Guidance	
11	(i)		(OP =) $100 - 20t$ (OQ =) $400 - 25t$	B1 B1	isw isw	Ignore labels
				2		
	(ii)		$(x^2) = (100-20t)^2 + (400-25t)^2$ $x^2 = 170000 - 24000t + 1025t^2$	M1 A1 A1	Use of Pythagoras on *their* expressions. Soi ignore lack of x^2 Final answer must include x^2	Condone use of $20t - 100$ etc for full marks
				3		
	(iii)		$\frac{\text{d}}{\text{d}t}(x^2) = -24000 + 2050t$ $= 0$ when $t = \frac{24000}{2050}$ $\left(= \frac{480}{41}\right) = (11.7)$ **oe** Then $x^2 = 29512$ $\Rightarrow x = 172\,(\text{m})$	M1 A1 M1dep A1 M1dep A1	Diffn of *their* fn : reduction of power in at least one term Correct numerical expression isw Set = 0 and attempt to solve Allow correct answer even if premature division in (i) Substitute *their* t(providing $t > 0$). Dep on both M	Ignore incorrect constant from (ii) Ignore notation on lhs SC 1 for $b + 2ct$
				6		
	(iv)		Car takes 5 secs to reach O Train takes 16 secs	B1	Numerical evidence for both required	Accept other valid explanations
				1		

Question			Answer	Marks	Guidance	
12	(i)		$x + 3y = k$ or $y = -\frac{1}{3}x + c$ or $\frac{y-b}{x-a} = -\frac{1}{3}$	M1		
			substitute (8, 3)	M1dep		
			gives $x + 3y = 17$ **oe**	A1	3 term equation isw	$k = 17$ or $c = \frac{17}{3}$ $y = -\frac{1}{3}x + \frac{17}{3}$
				3		
	(ii)		Solve *their* L_2 with $y = 3x - 1$ simultaneously: $x = 2$, $y = 5$	M1 A1 A1	Must lead to a value for x or y	SC3 Checking points and finding that (2, 5) lies on both
				3		
	(iii)		$d^2 = (8-2)^2 + (3-5)^2 \quad (=40)$	M1	Application of Pythagoras	
			$\Rightarrow d = \sqrt{40} \quad \left(= 2\sqrt{10} = 6.32\right)$	A1		
				2		
	(iv)		$(x-8)^2 + (y-3)^2 = 40$	B1	FT from (iii) Allow 6.32^2 oe	
				1		
	(v)		The point is on the other end of the diameter: (2, 5) to (8, 3) is $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$	M1	Using (8, 3) and *their* Q from (ii).	Alternatively: $\frac{2+x}{2} = 8, \quad \frac{5+y}{2} = 3$ M1 $\Rightarrow x = 14, \ y = 1$ A1
			$\Rightarrow (14,1)$	A1		
			$3x - y = c$ satisfied by (14, 1) $\Rightarrow 3x - y = 41$ **oe**	A1	Only 3 terms	
				3		

Question			Answer	Marks	Guidance	
13	(i)		$\frac{540}{x}, \frac{540}{x+75}$ **oe**	**B1** **B1**	Condone $\frac{5.4}{x}, \frac{5.40}{x+0.75}$ or $\frac{5.40}{x+75}$	Ignore any labels. Allow $n \leqslant \frac{540}{x}$ etc
				2		
	(ii)		$\Rightarrow \frac{540}{x} = \frac{540}{x+75} + 5$ **oe** $\Rightarrow 540(x+75) = 540x + 5x(x+75)$ **oe** $\Rightarrow (540 \times 75 = 5x(x+75))$ $\Rightarrow x^2 + 75x - 8100 = 0$	**M1** **A1** **M1** **A1** **A1**	For forming 3 term eqn using *their* terms from (i) Condone –5 Correct eqn Clear both fractions. Eqn must have 3 terms with x and x ±75 involved in denominator for 2 terms AG. At least 1 intermediate step must be seen	May start again Any wrong algebra gets final A0
				5		
	(iii)		$x^2 + 75x - 8100 = 0$ $\Rightarrow (x-60)(x+135) = 0$ $\Rightarrow x = 60$ $\Rightarrow x + 75 (= 135)$ or $60 + 75$ Buns 60p, loaf of bread 135p **oe**	**M1** **A1** **A1** **A1** **A1**	Solving given quadratic by factorisation that would lead to 2 terms correct when expanded soi Or correct formula soi Correct factorisation or correct substitution soi by final answer cao www - units must be given	Ignore –135 Correct answer only full marks
				5		

Question			Answer	Marks	Guidance	
14	(i)		$\Rightarrow 7 = (-3)^3 + (-3)^2 a - 3b + 1$ **oe** and $3 = 1 + a + b + 1$ **oe** $\Rightarrow \left(9a - 3b = 33 \text{ and } a + b = 1\right)$ $\Rightarrow a = 3,\ b = -2$	**B1** **B1** **M1** **A1**	1st equation 2nd equation Solve *their* eqns leading to either *a* or *b* Both **AG**	Need not be simplified for either Need to see at least one intermediate step
				4		
	(ii)		Midpoint is (–1, 5) Show (–1, 5) lies on curve.	**B1** **B1**	 Must see –1 + 3 + 2 + 1 = 5	 i.e. powers must be evaluated
				2		
	(iii)		$A_1 = \pm\left(\text{Area under curve} - \text{area under line}\right)$ or $A_2 = \pm\left(\text{Area under line} - \text{area under curve}\right)$ Area under curve $= \int\left(x^3 + 3x^2 - 2x + 1\right)dx = \frac{x^4}{4} + x^3 - x^2 + x\left(+c\right)$ $A_1 = \left(\left(\frac{1}{4} - 1 - 1 - 1\right) - \left(\frac{81}{4} - 27 - 9 - 3\right)\right) - 12$ $= \left(-\frac{11}{4} - -\frac{75}{4}\right) - 12 = 16 - 12 = 4$ $A_2 = 8 - \left(\left(\frac{1}{4} + 1 - 1 + 1\right) - \left(\frac{1}{4} - 1 - 1 - 1\right)\right) = 8 - 4 = 4$	**B1** **M1** **A1** **M1dep** **A1** **A1**	For sight of one attempt to find a difference of areas For integration, ignore limits Integration correct Correct limits for one curve integral (For A_1, –3 to –1 and for A_2, –1 to 1) For A_1 www For A_2 www	 At least 3 powers increased by 1. Watch for multiplication by *x* Could be wrong way round but must be subtracted n.b. an answer of –4 should be explained for credit of A1
				6		

Question			Answer	Marks	Guidance	
			Alternative 1: if subtraction is before integration.	B1	For subtracting *their* $y = 4 - x$ from curve	Could be subtracted in either order
			$(\text{A}_1)\ = \int\left(x^3+3x^2-2x+1-(4-x)\right)\mathrm{d}x = \frac{x^4}{4}+x^3-\frac{x^2}{2}-3x(+c)$	M1	For either integration, ignore limits	
			$= \left(\left(\frac{7}{4}\right)-\left(-\frac{9}{4}\right)\right)=4$	A1	Either integration correct	
			$(\text{A}_2)\ = \int\left((4-x)-(x^3+3x^2-2x+1)\right)\mathrm{d}x = -\frac{x^4}{4}-x^3+\frac{x^2}{2}+3x(+c)$	M1dep	Correct limits for one curve integral (For A_1, –3 to –1 and for A_2, –1 to 1)	Could be wrong way round but must be subtracted.
			$= \left(\left(\frac{9}{4}\right)-\left(-\frac{7}{4}\right)\right)=4$	A1	For A_1 www	n.b. an answer of –4 should be explained for credit of A1
				A1	For A_2 www	
			Alternative 2	B1	For subtracting *their* $y = 4 - x$ from curve	
			$y=\left(x^3+3x^2-2x+1\right)-(4-x)=x^3+3x^2-x-3$			
			$y=(x+1)^3-4(x+1)$	M1 A1	Writing as a function of $(x + 1)$	
			This is an odd function relative to $x = -1$. The function therefore has 180^0 rotational symmetry about (–1, 0)	M1 A1	Understanding of odd function	
			So $\text{A}_1 = \text{A}_2$	A1	Rotational symmetry Conclusion	

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