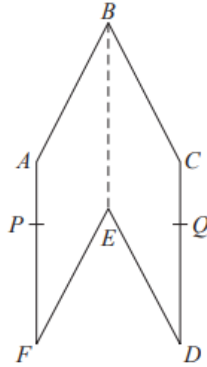


An Edexcel GCSE Maths Question

22 The diagram shows a hexagon $ABCDEF$.

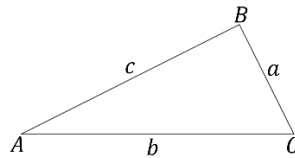


$ABEF$ and $CBED$ are congruent parallelograms where $AB = BC = x$ cm.
 P is the point on AF and Q is the point on CD such that $BP = BQ = 10$ cm.

Given that angle $ABC = 30^\circ$,

prove that $\cos PBQ = 1 - \frac{(2 - \sqrt{3})}{200}x^2$

Cosine rule:



$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{or} \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Using the cosine rule, in the first form, on triangle ABC shows that $AC^2 = 2x^2 - 2x^2 \cos 30$

You are expected to know, or be able to work out, that $\cos 30^\circ = \frac{\sqrt{3}}{2}$.

Using this, you can see that

$$\begin{aligned} AC^2 &= 2x^2 - 2x^2 \times \frac{\sqrt{3}}{2} \\ &= 2x^2 - \sqrt{3}x^2 \\ &= x^2(2 - \sqrt{3}). \end{aligned}$$

Using the cosine rule on triangle PBQ shows that $PQ^2 = 200 - 200 \cos PBQ$.

Making $\cos PBQ$ the subject gives $\cos PBQ = \frac{200 - PQ^2}{200} = 1 - \frac{PQ^2}{200}$.

$AC = PQ$ therefore $\cos PBQ = 1 - \frac{AC^2}{200} = 1 - \frac{(2 - \sqrt{3})}{200}x^2$.