

Further Pure 1 Taylor Series

Given that $f(x)$ satisfies the differential equation $f'(x) = y - x^3$ and that $f(1) = 2$, find a series solution for $f(x)$ in ascending powers of $x - 1$ up to an including the term in $(x - 1)^4$.

$$f'(1) = 1$$

$$f''(x) = f'(x) - 3x^2$$

$$f''(1) = -2$$

$$f'''(x) = f''(x) - 6x$$

$$f'''(1) = -8$$

$$f^{(4)}(x) = f'''(x) - 6$$

$$f^{(4)}(1) = -14$$

$$f(x) \approx 2 + (x - 1) - (x - 1)^2 - \frac{4}{3}(x - 1)^3 - \frac{7}{12}(x - 1)^4$$