

Further Pure 1 Numerical Methods

Use the approximations $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$ and $\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$ to obtain estimates for y_1, y_2 and y_3 for the differential equation $\frac{d^2y}{dx^2} = x^2 + y - 1$, given that when $x = 2$, $y = 3$ and $\frac{dy}{dx} = 1$, $h = 0.01$.

$$y_1 \approx y_0 + h \left(\frac{dy}{dx}\right)_0 = 3 + 0.01 \times 1 = 3.01$$

$$y_2 \approx h^2 \left(\frac{d^2y}{dx^2}\right)_1 + 2y_1 - y_0 \approx 0.01^2(2.01^2 + 3.01 - 1) + 2 \times 3.01 - 3 = 3.02060501$$

$$y_3 \approx h^2 \left(\frac{d^2y}{dx^2}\right)_2 + 2y_2 - y_1 \approx 0.01^2(2.02^2 + 3.02060501 - 1) + 2 \times 3.02060501 - 3.01$$

$$= 3.041820121$$

$$y_1 = 3.01$$

$$y_2 = 3.02060501$$

$$y_3 = 3.041820121$$

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