<u>Core Pure 2 Volumes of Revolution</u>

A curve is defined parametrically by the equations $x = \tan\theta$ and $y = \sec^3\theta$, $\quad 0 \le \theta < \frac{\pi}{2}$.

The region R bounded by the curve, the y axis and the line $y = 8$ is rotated through 2π radians about the y axis. Find the volume of the solid of revolution formed.

The curve intersect the y axis when $\tan\theta = 0$.

$$\tan\theta = 0 \Rightarrow \theta = 0 \Rightarrow y = 1$$

When $y = 8, \cos\theta = \frac{1}{2}$ *and* $\theta = \frac{\pi}{3}$

$$\frac{dy}{d\theta} = 3\sec^3\theta\tan\theta$$

The required volume is

$$\pi\int_1^8 x^2 dy = 3\pi\int_0^{\frac{\pi}{3}} \tan^3\theta\sec^3\theta \; d\theta$$

$$= 3\pi\int_0^{\frac{\pi}{3}} (\sec^2\theta - 1)\tan\theta\sec^3\theta \, d\theta$$

$$= 3\pi\int_0^{\frac{\pi}{3}} \sec^5\theta\tan\theta - \sec^3\theta\tan\theta \; d\theta$$

$$= 3\pi\left[\frac{1}{5}\sec^5\theta - \frac{1}{3}\sec^3\theta\right]_0^{\frac{\pi}{3}}$$

$$= 3\pi\left(\frac{32}{5} - \frac{8}{3}\right) - 3\pi\left(\frac{1}{5} - \frac{1}{3}\right)$$

$$= \frac{58\pi}{5}$$

Bury Maths Tutor