

Core Pure 2 Hyperbolic Functions

Find $\int \frac{1}{\sqrt{x^2 + 4x + 8}} dx$

$$\begin{aligned} I &= \int \frac{1}{\sqrt{x^2 + 4x + 8}} dx \\ &= \int \frac{1}{\sqrt{(x+2)^2 + 4}} dx \end{aligned}$$

Using the substitution $x + 2 = 2 \sinh u$, $dx = 2 \cosh u du$ and

$$\begin{aligned} I &= \int \frac{2 \cosh u}{\sqrt{4 \sinh^2 u + 4}} du \\ &= \int \frac{2 \cosh u}{\sqrt{4 \cosh^2 u}} du \\ &= \int 1 du \\ &= u \\ &= \operatorname{arcsinh}\left(\frac{x+2}{2}\right) + C \end{aligned}$$

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