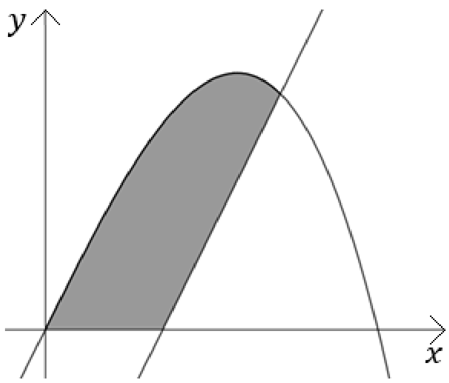
Core Pure 1 Volume of Revolution

The shaded region shown below formed by the line with equation $y = 2x - 1$ the curve with equation $y = 2x - x^3$ and the $x$ axis is rotated $2\pi$ radians about the $x$ axis. Find the volume of the solid formed.




The line and curve intersect where $2x - x^3 = 2x - 1$. That is where $x = 1$ and $y = 1$.

The line intersects the $x$ axis at $x = \frac{1}{2}$.

The volume can be found by subtracting the volume of a cone from the appropriate integral.

$$V = \pi \int_0^1 (2x - x^3)^2 dx - \frac{1}{3}\pi \times 1^2 \times \frac{1}{2}$$

$$= \pi \left( \int_0^1 4x^2 - 4x^4 + x^6 \, dx - \frac{1}{6} \right)$$

$$= \pi \left( \left[ \frac{4}{3}x^3 - \frac{4}{5}x^5 + \frac{1}{7}x^7 \right]_0^1 - \frac{1}{6} \right)$$

$$= \pi \left( \frac{4}{3} - \frac{4}{5} + \frac{1}{7} - \frac{1}{6} \right)$$

$$= \pi \frac{280 - 168 + 30 - 35}{210}$$

$$= \frac{107}{210}\pi$$

Bury Maths Tutor