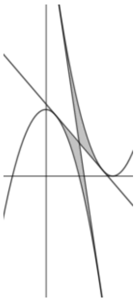
The diagram shows the parabolas with equations $y = 4 - x^2$ and $y = (x - 4)^2$ together with the straight lines that are tangents to both parabolas.

Find the area enclosed by the parabolas and the tangents, shown shaded in the diagram.



By symmetry, the point of intersection of the tangents is $(2,2)$.

$$y = 4 - x^2 \Rightarrow \frac{dy}{dx} = -2x$$

$(a, 4 - a^2)$ and $(2,2)$ are points on a tangent with gradient $-2a$.

$$-2a = \frac{2-4+a^2}{2-a} \Rightarrow 2a^2 - 4a = a^2 - 2 \Rightarrow a^2 - 4a + 2 = 0 \Rightarrow a = 2 \pm \sqrt{2}$$

The gradients are $-4 \pm 2\sqrt{2}$

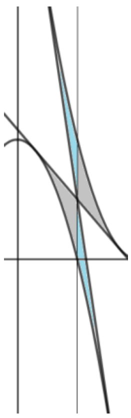
The tangents are $y - 2 = (-4 \pm 2\sqrt{2})(x - 2)$

$y = \left(-4 + 2\sqrt{2}\right)x + 10 - 4\sqrt{2}$ and $y = \left(-4 - 2\sqrt{2}\right)x + 10 + 4\sqrt{2}$

The required area is

$$2\int_{2-\sqrt{2}}^{2} \left(\left(-4 + 2\sqrt{2}\right)x + 10 - 4\sqrt{2} - (4 - x^2)\right) + \left((x-4)^2 - \left(\left(-4 - 2\sqrt{2}\right)x + 10 + 4\sqrt{2}\right)\right) dx$$

$$= \frac{8\sqrt{2}}{3}$$



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