The equation $x^2 - 2x + 10 = 0$ has roots α and β.

Find a quadratic equation with roots α^3 and β^3.

Substitution Method

An equation with the required roots is $\left(u^{\frac{1}{3}}\right)^2 - 2u^{\frac{1}{3}} + 10 = 0$

$$u^{\frac{1}{3}}\left(u^{\frac{1}{3}} - 2\right) = -10$$

$$u\left(u - 6u^{\frac{2}{3}} + 12u^{\frac{1}{3}} - 8\right) = -1000$$

$$u\left(u - 6\left(u^{\frac{2}{3}} - 2u^{\frac{1}{3}} + 10\right) + 52\right) = -1000$$

$$u^2 + 52u + 1000 = 0$$

Sums and Products of Roots Method

$$\alpha + \beta = 2 \qquad \alpha\beta = 10$$

$$(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 = 8$$

$$\Rightarrow \alpha^3 + \beta^3 = 8 - 3\alpha\beta(\alpha + \beta) \ = 8 - 3 \times 10 \times 2 = -52$$

$$\alpha^3\beta^3 = 1000$$

The required equation is $u^2 + 52u + 1000 = 0$