

$$z = f(x, y) \quad x = g(s, t) \quad y = h(s, t)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t}$$

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial}{\partial t} \left(\frac{\partial z}{\partial x} \right) \frac{\partial x}{\partial t} + \frac{\partial z}{\partial x} \frac{\partial^2 x}{\partial t^2} + \frac{\partial}{\partial t} \left(\frac{\partial z}{\partial y} \right) \frac{\partial y}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial^2 y}{\partial t^2}$$

$$= \left(\frac{\partial}{\partial x} \left(\frac{\partial z}{\partial x} \right) \frac{\partial x}{\partial t} + \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x} \right) \frac{\partial y}{\partial t} \right) \frac{\partial x}{\partial t} + \frac{\partial z}{\partial x} \frac{\partial^2 x}{\partial t^2} + \left(\frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y} \right) \frac{\partial x}{\partial t} + \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial y} \right) \frac{\partial y}{\partial t} \right) \frac{\partial y}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial^2 y}{\partial t^2}$$

$$= \frac{\partial^2 z}{\partial x^2} \left(\frac{\partial x}{\partial t} \right)^2 + \frac{\partial^2 z}{\partial y \partial x} \frac{\partial y}{\partial t} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial x} \frac{\partial^2 x}{\partial t^2} + \frac{\partial^2 z}{\partial x \partial y} \frac{\partial x}{\partial t} \frac{\partial y}{\partial t} + \frac{\partial^2 z}{\partial y^2} \left(\frac{\partial y}{\partial t} \right)^2 + \frac{\partial z}{\partial y} \frac{\partial^2 y}{\partial t^2}$$

$$= \frac{\partial^2 z}{\partial x^2} \left(\frac{\partial x}{\partial t} \right)^2 + 2 \frac{\partial^2 z}{\partial x \partial y} \frac{\partial x}{\partial t} \frac{\partial y}{\partial t} + \frac{\partial^2 z}{\partial y^2} \left(\frac{\partial y}{\partial t} \right)^2 + \frac{\partial z}{\partial x} \frac{\partial^2 x}{\partial t^2} + \frac{\partial z}{\partial y} \frac{\partial^2 y}{\partial t^2}$$