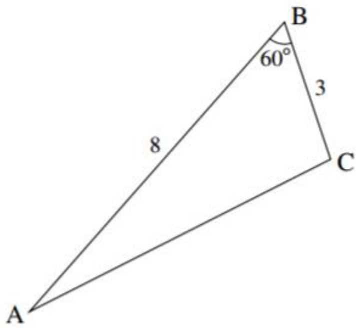
OCR Additional Maths Exam Questions - Trigonometry

1 The angle θ is greater than 90° and less than 360° and $\cos\theta = \frac{2}{3}$. Find the exact value of $\tan\theta$. **[3]**

7 The course of a cross-country race is in the shape of a triangle ABC.
AB = 8 km, BC = 3 km and angle ABC = 60°.



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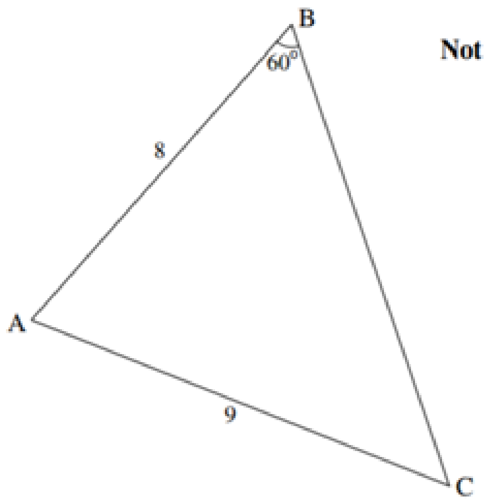
(i) Calculate the distance AC and hence the total length of the course. **[4]**

(ii) The organisers extend the course so that AC = 9 km.



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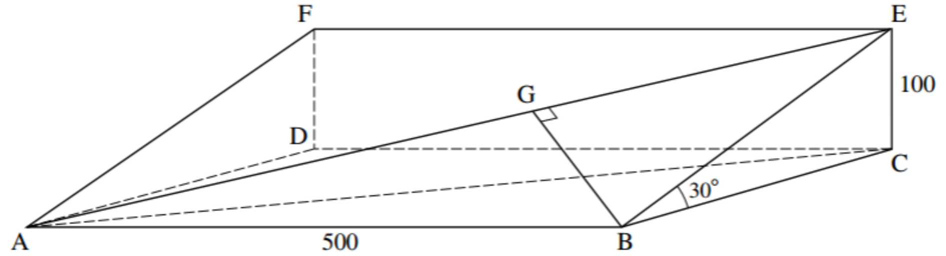
Calculate the angle BCA. **[3]**

12 The diagram shows a rectangle ABEF on a plane hillside which slopes at an angle of 30° to the horizontal. ABCD is a horizontal rectangle. E and F are 100 m vertically above C and D respectively. AB = DC = FE = 500 m.

AE is a straight path.
From B there is a straight path which runs at right angles to AE, meeting it at G.



(i) Find the distance BE. **[3]**

(ii) Find the angle that the path AE makes with the horizontal. **[4]**

(iii) Find the area of the triangle ABE.

Hence find the length BG. **[5]**

7 It is required to solve the equation $\sin\theta\cos\theta = \frac{1}{4}$.

(i) Show that $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\sin\theta\cos\theta}$. **[1]**

(ii) Hence show that the equation $\sin\theta\cos\theta = \frac{1}{4}$ is equivalent to $\tan\theta + \dfrac{1}{\tan\theta} = 4$. **[2]**

(iii) By expressing this equation as a quadratic equation in t, where $t = \tan\theta$, find the two values of θ, in the range $0° \leqslant \theta \leqslant 180°$, that satisfy the equation. **[4]**

11 Michael is at a point A and the base of a church tower is at a point F, as shown in Fig. 11.
He measures the bearing of the tower to be 060°.
Michael walks 100 metres due North to the point B from where he measures the bearing of F to be 110°.
The triangle ABF is in the horizontal plane.

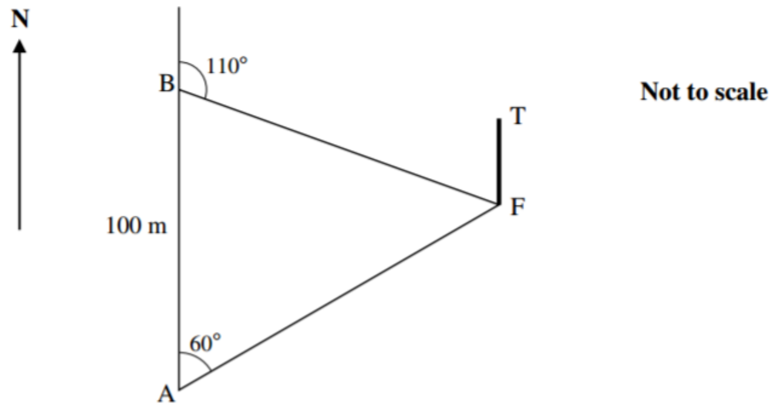


Fig. 11

(i) Show that AF = 122.7 m, correct to 4 significant figures, and find BF. **[5]**

Michael finds that the angle of elevation of the top of the tower, T, from A is 10°.

(ii) Find the height of the tower. **[2]**

C is the point on AB that is nearest to F.

(iii) Find CF and the angle of elevation from C to the top of the tower, correct to 1 decimal place. **[5]**

3 In the triangle PQR, PQ = 8 cm, RQ = 9 cm and RP = 7 cm.

(i) Find the size of the largest angle. **[4]**

(ii) Calculate the area of the triangle. **[3]**

4 Solve the equation $5\sin 2x = 2\cos 2x$ in the interval $0° \leqslant x \leqslant 360°$.
Give your answers correct to 1 decimal place. **[5]**

10 You are given that $\sin\theta = \frac{2}{5}$ with $0° \leqslant \theta \leqslant 90°$.

Using the identity $\sin^2\theta + \cos^2\theta = 1$, find an exact value for $\cos\theta$. **[3]**

4 You are given that θ is an acute angle and $\sin\theta = \frac{\sqrt{5}}{3}$.

Find the **exact** value of $\tan\theta$. **[3]**

13 A pyramid has a square base, ABCD, with vertex E. E is directly above the centre of the base, O, as shown in Fig. 13.
The lengths of the sides of the base are each $2x$ metres and the height is h metres.
The lengths of the sloping edges, AE, BE, CE and DE, are each 5 metres.

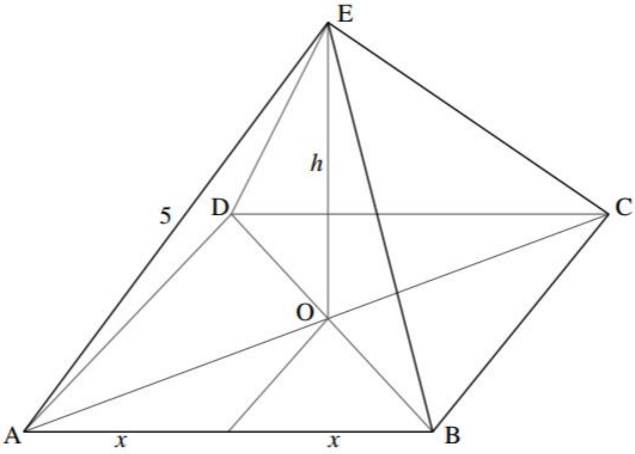


Fig. 13

(i) Show that $2x^2 = 25 - h^2$. **[2]**

(ii) Show that the volume of the pyramid, V m^3, is given by $V = \dfrac{50h - 2h^3}{3}$. **[2]**

(iii) As h varies, find the value of h for which V has a stationary value. **[4]**

(iv) Prove that this stationary value is a maximum. **[2]**

(v) Calculate the angle between the edge AE and the base when h takes this value. **[2]**

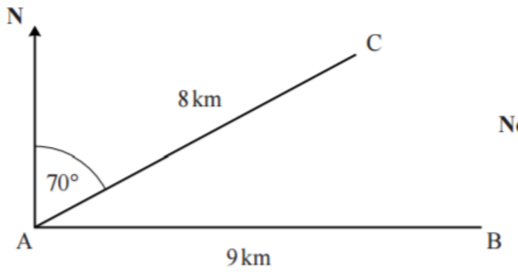
[Volume of a pyramid = $\frac{1}{3}$ × base area × height.]

5 **(i)** Show that the equation $3\cos^2\theta = \sin\theta + 1$ can be written as $3\sin^2\theta + \sin\theta - 2 = 0$. **[2]**

(ii) Solve this equation to find values of θ in the range $0° < \theta < 360°$ that satisfy

$$3\cos^2\theta = \sin\theta + 1\ .$$ **[4]**

7 A yachtsman wishes to sail from a port, A, to another port, B, which is 9 km due East of A. Because of the wind he is unable to sail directly East and sails 8 km on a bearing of 070° to point C.



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Calculate

(i) the distance he is now from port B, **[3]**

(ii) the angle ABC and hence the bearing on which he must sail to reach port B from point C, correct to the nearest degree. **[4]**

9 The height above the ground of a seat on a fairground big wheel is h metres. At time t **minutes** after the wheel starts, h is given by

$$h = 7 - 5\cos(480t)°.$$

(i) Write down the initial height above the ground of the seat (when $t = 0$). **[1]**

(ii) Find the greatest height reached by the seat. **[2]**

(iii) Calculate the time of the first occasion when the seat is 9 metres above the ground. Give your answer correct to the nearest second. **[4]**

7 John and Jennie are asked to draw a triangle ABC with the following properties:

AC = 6 cm, CB = 4 cm and the angle A = 40°.

John draws the triangle as shown in Fig. 7.1 and Jennie draws the triangle as shown in Fig. 7.2.

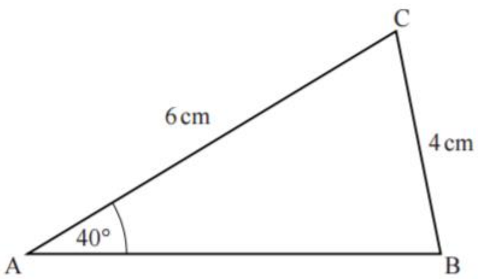

Fig. 7.1

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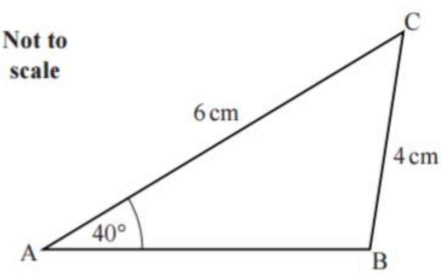
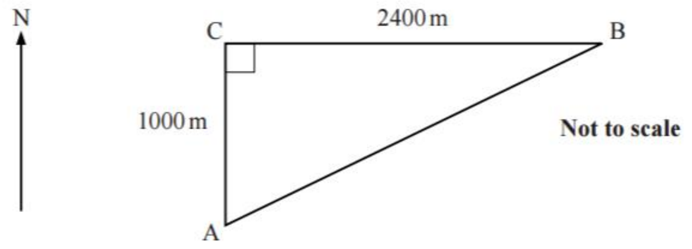


Fig. 7.2

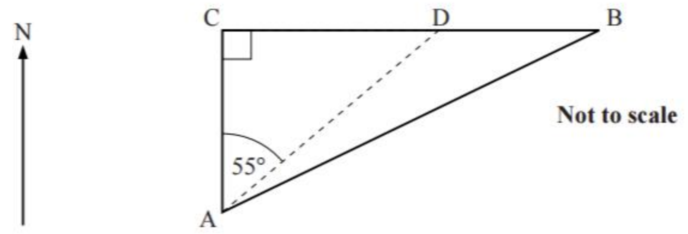
Calculate the angle B in each case. **[4]**

10 One leg of a cross-country race is from A to B. The checkpoint B is at the end of a wall that runs due east-west, as shown in the diagram. A is a point 1000 m due south of a point C on the wall. BC = 2400 m.



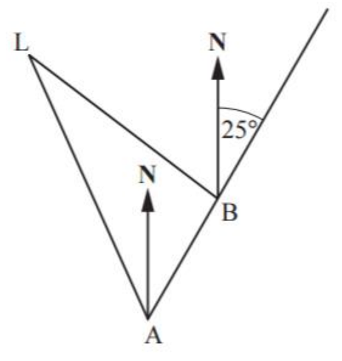
(i) What bearing should a runner take to travel from A to B and what is the distance AB? **[4]**

John sets off from A unable to see the checkpoint, B. He heads out on a bearing of 055° and when he reaches the wall at point D he knows he has to go east along the wall to reach the point B, as shown in the diagram.



(ii) How much further than the distance AB does John run? **[3]**

5 A ship is moving on a bearing of 025° at 14 knots (1 knot = 1 nautical mile per hour). As it passes point A, a lighthouse L is seen on a bearing of 340°. After 30 minutes, the ship passes point B from where the lighthouse is seen on a bearing of 320°.



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(i) Find the angle BAL and the angle ALB. **[3]**

(ii) Hence, or otherwise, calculate the distance BL in nautical miles. **[3]**

9 **(i)** Show that $\frac{1-\cos^2 x}{1-\sin^2 x} = \tan^2 x$. **[1]**

(ii) Hence solve the equation $\frac{1-\cos^2 x}{1-\sin^2 x} = 3 - 2\tan x$ for values of x in the range $0° \leqslant x \leqslant 180°$. **[4]**

2 **(i)** Find α in the range $0° \leqslant \alpha \leqslant 180°$ such that $\tan\alpha = -1.5$. **[2]**

(ii) Find β in the range $0° \leqslant \beta \leqslant 180°$ such that $\sin\beta = 0.2$. **[2]**

10 Fig. 10 shows a partly open window OA, viewed from above. The window is hinged at O. When the window is closed, the end A is at point B. The window is kept open by a rod CD, where C is a fixed point on the line OB.
The point D slides along a fixed bar EF. When the window is closed, D is at F. When the window is fully open, D is at E.

$OA = OB = 20\,\text{cm}$, $OC = 8\,\text{cm}$, $CD = 7\,\text{cm}$, $EF = 5\,\text{cm}$, $OE = 10\,\text{cm}$

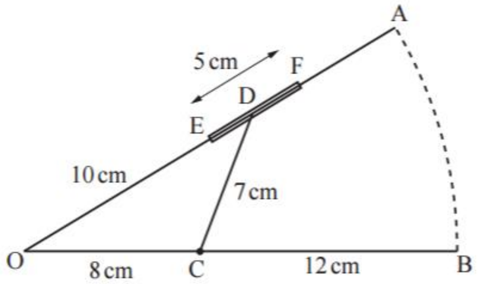


Fig. 10

Find

(i) angle EOC when the window is fully open, **[3]**

(ii) the distance OD when angle EOC is 30°. **[4]**

13 A gardener marks out a regular hexagon ABCDEF on his horizontal garden.
Each side of the hexagon is 0.5 m. The gardener sticks a cane in the ground at each point of the hexagon. He joins the six canes at V where V is vertically above the centre, O, of the hexagon, as shown in Fig. 13. Each cane has a length of 2.4 m from the ground to V.

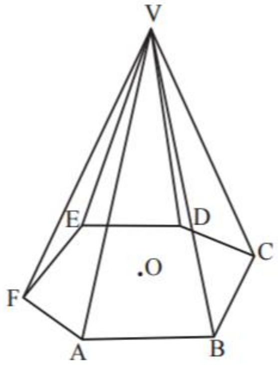


Fig. 13

Calculate, giving your answers to 3 significant figures,

(i) the vertical height of V above the ground, **[3]**

(ii) the angle between each cane and the ground, **[2]**

(iii) the angle between the plane VAB and the ground. **[4]**

The gardener stretches a horizontal wire around the structure to strengthen it. He fixes the wire to each cane at a point 1 m vertically above the ground.

(iv) Find the length of the wire. **[3]**

3 A triangle has sides 8 cm, 7 cm and 12 cm. Calculate the largest angle of the triangle, correct to the nearest degree. [5]

4 Find the values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying the equation

$$4\sin\theta = 3\cos\theta.$$

Give your answers to the nearest 0.1 degree. [4]

9 **(i)** Using the identity $\cos^2\theta + \sin^2\theta = 1$, show that the equation

$$2\cos^2\theta + \sin\theta = 2$$

can be written as $2\sin^2\theta - \sin\theta = 0$. [2]

(ii) Hence find all values of θ in the range $0° \leqslant \theta \leqslant 180°$ satisfying the equation

$$2\cos^2\theta + \sin\theta = 2.$$ [4]

2 Adam and Beth set out walking from a point P. After one hour Adam is 3.6 kilometres due north of P and Beth is 2.5 kilometres from P on a bearing of 035°.

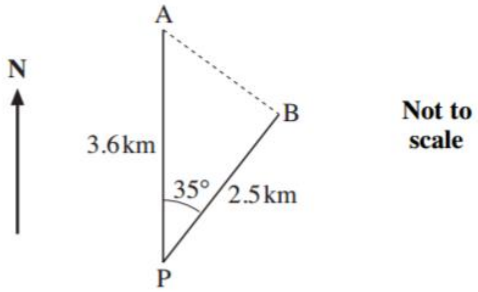


Fig. 2

Calculate how far they are apart at this time. Give your answer correct to 2 significant figures.[4]

3 Calculate the values of x in the range $0° < x < 360°$ for which $\sin x = 2\cos x$. [4]

13 Fig. 13.1 shows a solid block which is in the shape of a pyramid. The horizontal base, ABCD, is a square with side 20 cm and the vertex, V, is 15 cm vertically above the centre, O, of the square base. N is the midpoint of AB.

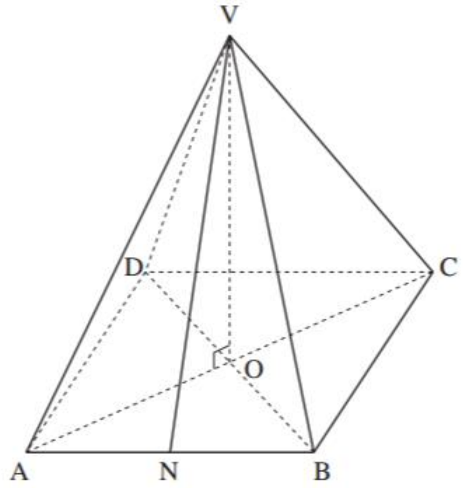


Fig. 13.1

(i) Calculate the length of the diagonal AC. [2]

(ii) Show that the length of the edge AV is $\sqrt{425}$ cm. [2]

(iii) Calculate the angle that the edge AV makes with the base. [2]

(iv) Calculate the length VN. [2]

M is the point on VB such that AM is perpendicular to VB as shown in Fig. 13.2.

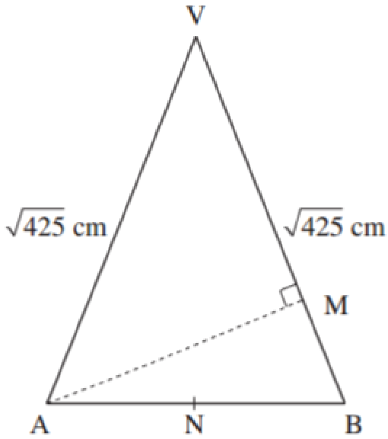
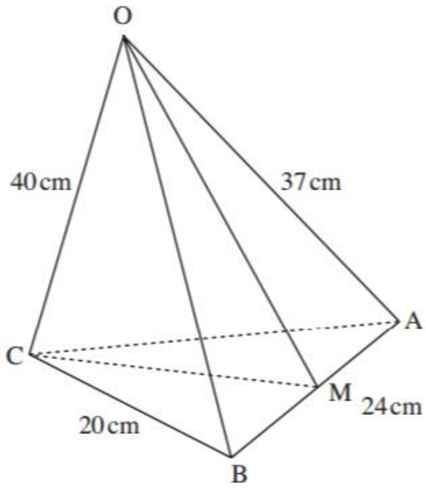


Fig 13.2

(v) Calculate the area of triangle VAB. Hence or otherwise calculate the distance AM. [4]

4 Find all the values of x in the range $0° < x < 360°$ that satisfy $\sin x = -4\cos x$. [5]

13 In the pyramid OABC, OA = OB = 37 cm, OC = 40 cm, CA = CB = 20 cm and AB = 24 cm. M is the midpoint of AB.



Calculate

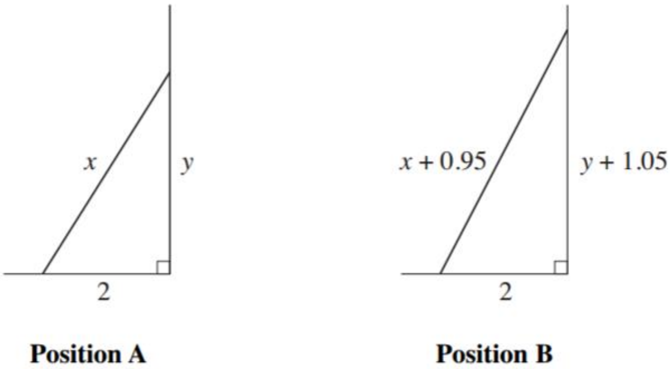
(i) the lengths OM and CM, [3]

(ii) the angle between the line OC and the plane ABC, [4]

(iii) the volume of the pyramid. [5]

[The volume of a pyramid = $\frac{1}{3}$ × base area × height.]

14 An extending ladder has two positions. In position **A** the length of the ladder is x metres and, when the foot of the ladder is placed 2 metres from the base of a vertical wall, the ladder reaches y metres up the wall.



In position B the ladder is extended by 0.95 metres and it reaches an extra 1.05 metres up the wall.

The foot of the ladder remains 2 m from the base of the wall.

(i) Use Pythagoras' theorem for position **A** and position **B** to write down two equations in x and y. [2]

(ii) Hence show that $2.1y = 1.9x - 0.2$. [3]

(iii) Using these equations, form a quadratic equation in x.

Hence find the values of x and y. [7]

7 A pyramid stands on a horizontal triangular base, ABC, as shown in Fig. 7.
The angles CAB and ABC are 50° and 60° respectively.
The vertex, V, is directly above C with VC = 10 m.
The angle which the edge VA makes with the vertical is 40°.

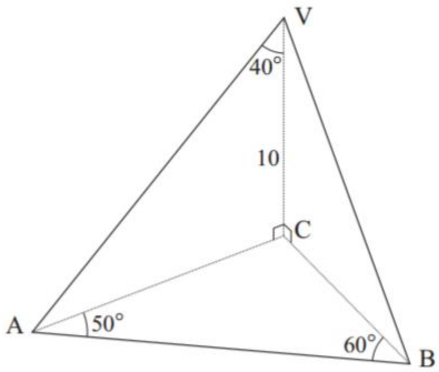


Fig. 7

(i) Calculate AC. [2]

(ii) Hence calculate AB. [4]

8 It is required to solve the equation $2\cos^2 x = 5\sin x - 1$.

(i) Show that this equation may be written as $2\sin^2 x + 5\sin x - 3 = 0$. [2]

(ii) Hence solve the equation $2\cos^2 x = 5\sin x - 1$ for values of x in the range $0° \leqslant x \leqslant 360°$. [4]

13 In the triangle shown in Fig. 13, M is the midpoint of BC.

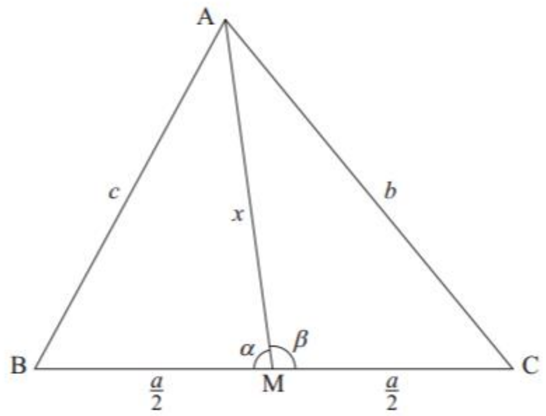


Fig. 13

(i) Explain why $\cos\alpha = -\cos\beta$. [2]

(ii) Using the cosine rule in the triangle BMA, show that

$$\cos\alpha = \frac{4x^2 + a^2 - 4c^2}{4ax}.$$ [2]

(iii) Find a similar expression for $\cos\beta$. [1]

(iv) Using the results in parts **(i)**, **(ii)** and **(iii)**, show that $4x^2 + a^2 = 2(c^2 + b^2)$. [5]

(v) A triangular lawn has sides 46 m, 29 m and 27 m. Find the distance from the midpoint of the longest side to the opposite corner. [2]