

STEP I 1991 Question 16

**16** At any instant the probability that it is safe to cross a busy road is 0.1. A toad is waiting to cross this road. Every minute she looks at the road. If it is safe, she will cross; if it is not safe, she will wait for a minute before attempting to cross again. Find the probability that she eventually crosses the road without mishap.

Later on, a frog is also trying to cross the same road. He also inspects the traffic at one minute intervals and crosses if it is safe. Being more impatient than the toad, he may also attempt to cross when it is not safe. The probability that he will attempt to cross when it is not safe is  $n/3$  if  $n \leq 3$ , where  $n$  minutes have elapsed since he first inspected the road. If he attempts to cross when it is not safe, he is run over with probability 0.8, but otherwise he reaches the other side safely. Find the probability that he eventually crosses the road without mishap.

What is the probability that both reptiles safely cross the road with the frog taking less time than the toad? If the frog has not arrived at the other side 2 minutes after he began his attempt to cross, what is the probability that the frog is run over (at some stage) in his attempt to cross?

[Once moving, the reptiles spend a negligible time on their attempt to cross the road.]