Roots of Polynomials - A Core Pure 2 Example

The equation $x^3 + 2x^2 + 3x + 4 = 0$ has roots a, b and c.

Without solving the equation find the value of $\frac{3}{a} + \frac{3}{b} + \frac{3}{c}$ and the value of $a^3 + b^3 + c^3$.

$$\left(\frac{3}{u}\right)^3 + 2\left(\frac{3}{u}\right)^2 + 3\left(\frac{3}{u}\right) + 4 = 0 \Rightarrow 27 + 18u + 9u^2 + 4u^3 = 0 \Rightarrow \frac{3}{a} + \frac{3}{b} + \frac{3}{c} = -\frac{9}{4}$$

$$(a + b + c)^3 \equiv a^3 + b^3 + c^3 + 3(a + b + c)(ab + bc + ac) - 3abc$$

$$a + b + c = -2 \qquad ab + bc + ac = 3 \qquad abc = -4$$

$$a^3 + b^3 + c^3 \equiv (a + b + c)^3 - 3(a + b + c)(ab + bc + ac) + 3abc$$

$$a^3 + b^2 + c^3 = (-2)^3 - 3(-2)(3) + 3(-4) = -2$$