

A company receives telephone calls at random at a mean rate of 2.5 per hour.

(a) Find the probability that the company receives

(i) at least 4 telephone calls in the next hour,

(ii) exactly 3 telephone calls in the next 15 minutes.

(b) Find to the nearest minute, the maximum length of time the telephone can be left unattended so that the probability of missing a call is less than 0.2.

The company puts an advert in the local newspaper. The number of calls received in a randomly selected two hour period after the paper is published is 10.

(c) Test at the 5% level of significance whether or not the mean rate of telephone calls has increased. State your hypotheses clearly.

(a)

(i) $X \sim Po(2.5)$

$$P(X \geq 4) = 0.2424$$

(ii) $X \sim Po\left(\frac{2.5}{4}\right)$

$$X \sim Po(0.625)$$

$$P(X = 3) = 0.0218$$

(b)

$$P(X = 0) > 0.8 \Rightarrow e^{-\lambda} > 0.8 \Rightarrow -\lambda > \ln 0.8 \Rightarrow \lambda < -\ln 0.8 = 0.2231$$

$$\frac{2.5}{60} = \frac{0.2231}{t} \Rightarrow t = \frac{60}{2.5} \times 0.2231 = 5.355 \approx 5 \text{ minutes.}$$

(c) H_0 : Calls are received at a rate of 2.5 per hour.

H_1 : Calls are received at a rate > 2.5 per hour.

According to the null hypothesis the mean number of calls received in a two hour period is 5.

$$X \sim Po(5)$$

$$P(X \geq 10 | \lambda = 5) = 0.0318 < 5\%$$

There is evidence at the 5% significance level that the rate has increased.