

- 7 A drug for treating a particular minor illness cures, on average, 78% of patients. Twenty people with this minor illness are selected at random and treated with the drug.
- (i) (A) Find the probability that exactly 19 patients are cured. [3]  
 (B) Find the probability that at most 18 patients are cured. [3]  
 (C) Find the expected number of patients who are cured. [1]
- (ii) A pharmaceutical company is trialling a new drug to treat this illness. Researchers at the company hope that a higher percentage of patients will be cured when given this new drug. Twenty patients are selected at random, and given the new drug. Of these, 19 are cured. Carry out a hypothesis test at the 1% significance level to investigate whether there is any evidence to suggest that the new drug is more effective than the old one. [8]
- (iii) If the researchers had chosen to carry out the hypothesis test at the 5% significance level, what would the result have been? Justify your answer. [2]

Let  $X$  be the number of patients cured.  $X \sim B(20, 0.78)$

(i)

$$(A) P(X = 19) = \binom{20}{19} \times 0.78^{19} \times 0.22 = 0.0392$$

$$(B) P(X \leq 18) = 1 - P(X \geq 19) = 1 - P(X = 19) - P(X = 20) = 1 - 0.0392 - 0.78^{20} = 0.954$$

$$(C) E(X) = np = 20 \times 0.78 = 15.6$$

(ii)

Let  $X \sim B(20, 0.78)$  and  $p$  = the probability that a patient is cured.

$$H_0: p = 0.78$$

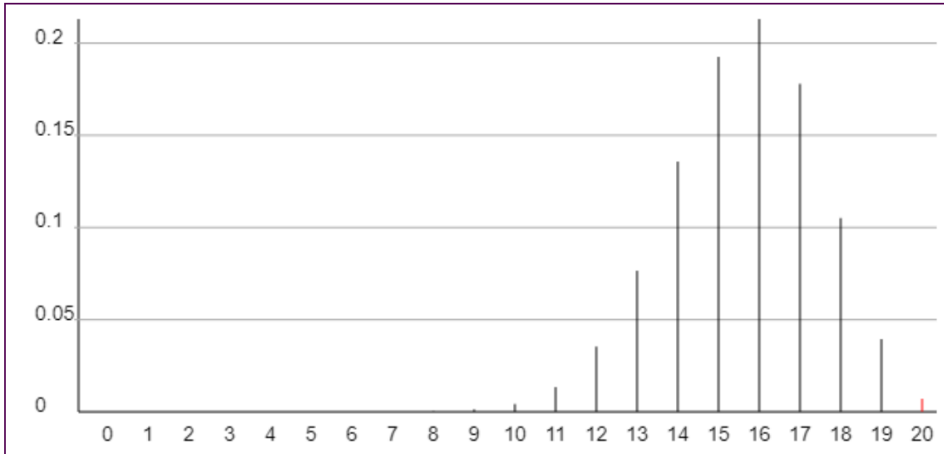
$$H_1: p > 0.78$$

$$P(X \geq 19) = 0.0392 + 0.78^{20} = 0.0461 > 1\%$$

There is insufficient evidence, at the 1% significance level, to conclude that the new drug is more effective than the old drug.

(iii)

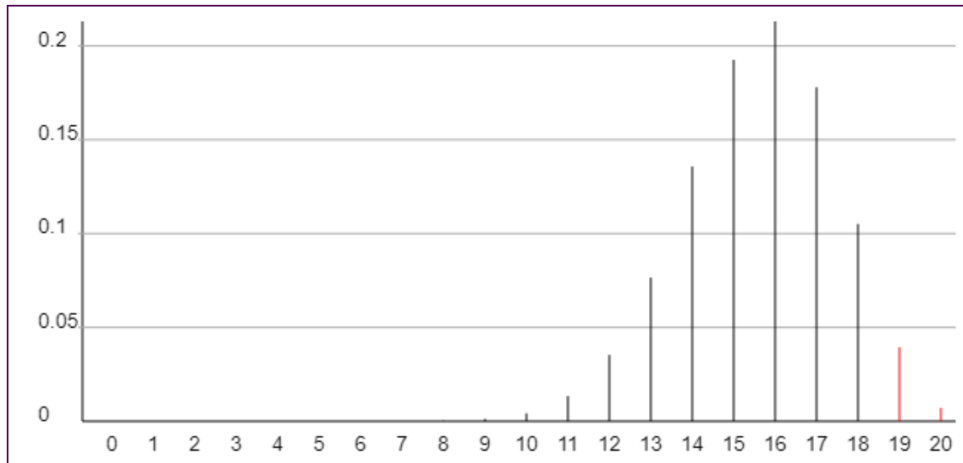
If the researchers had chosen to carry out the test at the 5% significance level, they would have concluded that there is sufficient evidence to reject  $H_0$  and claim that the new drug is better than the old drug because  $P(X \geq 19) = 0.046 < 5\%$ .



Upper tail  
 $P(X \geq 19) = 0.04615$   
 $P(X = 20) = 0.006949$

Actual significance level = 0.006949

Critical region:  $X = 20$



Upper tail  
 $P(X \geq 18) = 0.1512$   
 $P(X \geq 19) = 0.04615$

Actual significance level = 0.04615

Critical region:  $X \geq 19$