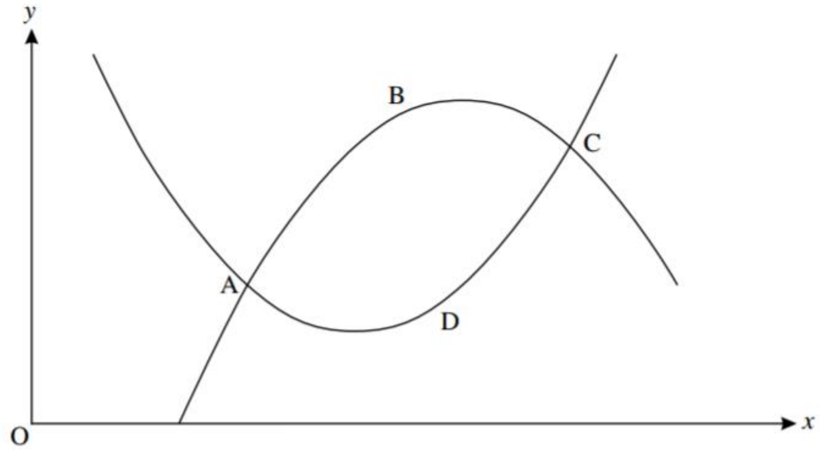
11 The shape ABCD below represents a leaf.
The curve ABC has equation $y = -x^2 + 8x - 9$.
The curve ADC has equation $y = x^2 - 6x + 11$.



(i) Find algebraically the coordinates of A and C, the points where the curves intersect. **[5]**

(ii) Find the area of the leaf. **[7]**

9 A car accelerates from rest. At time t seconds, its acceleration is given by $a = 4 - 0.2t \text{ m s}^{-2}$ until $t = 20$.

(i) Find the velocity after 5 seconds. **[3]**

(ii) What is happening to the velocity at $t = 20$? **[1]**

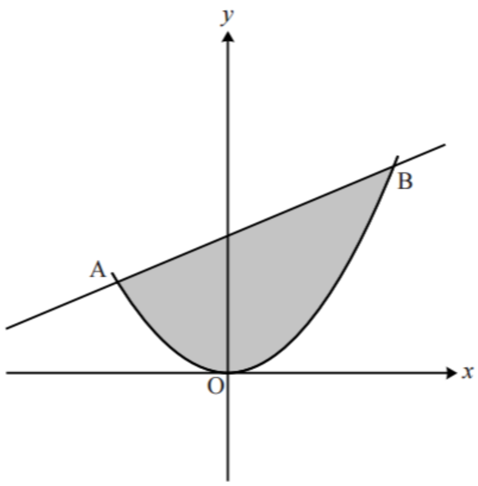
(iii) Find the distance travelled in the first 20 seconds. **[3]**

8 A train moves between two stations, taking 5 minutes for the journey.
The velocity of the train may be modelled by the equation $v = 60(t^4 - 10t^3 + 25t^2)$ where v is measured in metres per minute and t is measured in minutes.

Calculate the distance between the two stations. **[5]**

11 The shaded region in the diagram shows a wooden shape.

The curve has equation $y = \frac{1}{2}x^2$ and the coordinates of A are $(-2, 2)$.



The line AB is the normal to the curve at the point A.

(i) Find the equation of the line AB. **[5]**

(ii) Find the coordinates of the point B where the line AB meets the curve again. **[3]**

(iii) Find the shaded area. **[4]**

12 Fig. 12 shows the shape AOB that is to be made from card.

B is the point $(5, 0)$ and OB is part of the curve with equation $y = 0.3x^2 - 1.5x$.

The line AB is the normal to the curve at B.

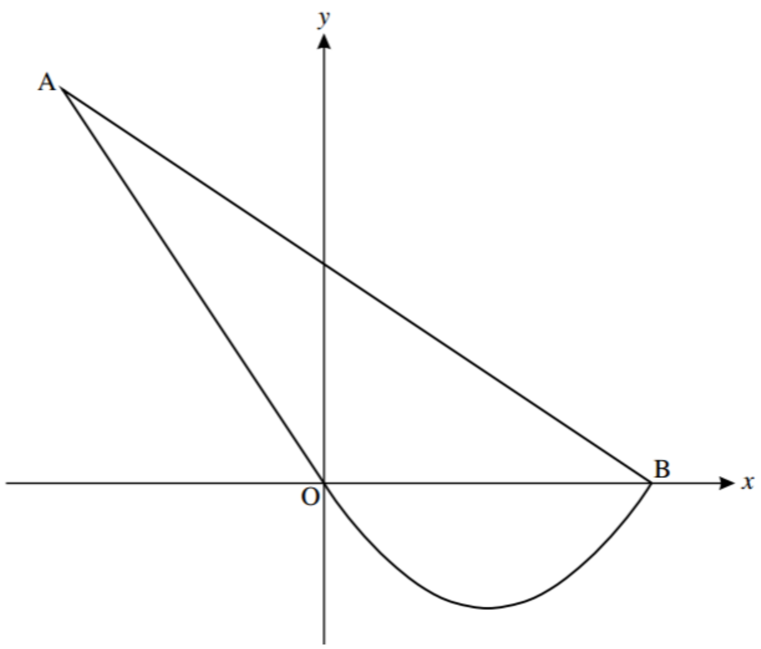


Fig. 12

(i) Find the equation of the line AB. **[4]**

The equation of the line AO is $2y + 3x = 0$.

(ii) Find the coordinates of the point A. **[3]**

(iii) Find the area of the shape AOB. **[5]**

9 The gradient function of a curve is given by $\frac{dy}{dx} = 3x^2 - 2x + 4$.

Find the equation of the curve, given that it passes through the point (2, 2). **[4]**

12 Two cars, A and B, move from rest away from a point O on a straight road starting at the same time.

(a) Car A moves with constant acceleration of $2\,\mathrm{m\,s^{-2}}$.

Express the displacement of car A after time t seconds as a function of t. **[2]**

(b) Car B moves with acceleration given by $a = \frac{1}{2}t + 1$.

Express the displacement of car B after time t seconds as a function of t. **[4]**

(c) **(i)** Find the time at which the cars are the same distance from O. **[2]**

(ii) Find the distance they have travelled at that time. **[2]**

(d) Draw a sketch graph of the velocity of each car on the axes given. **[2]**

14 The cross-section of a speed hump is modelled by the region enclosed by the x-axis and the curve

$$y = \frac{1 - (x - 1)^4}{5}.$$

The graph is shown in Fig. 14.
Units are metres.

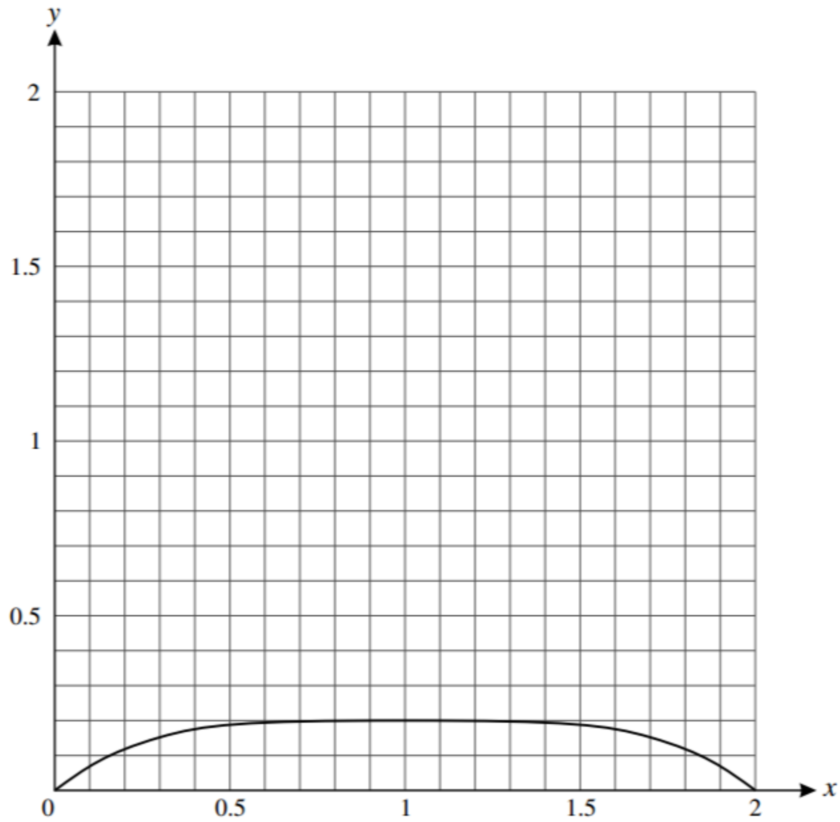


Fig. 14

(a) **(i)** Write down the maximum value of $1 - (x - 1)^4$. **[1]**

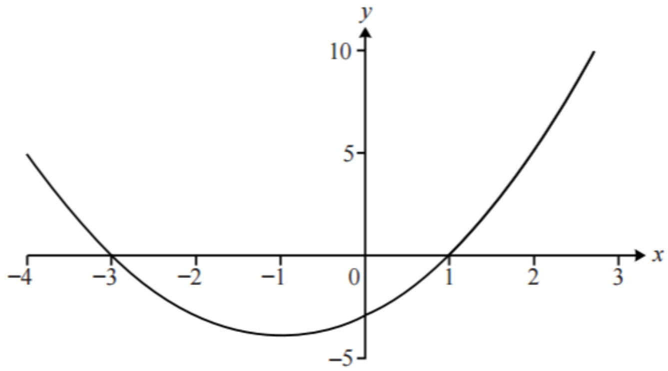
(ii) Hence write down the maximum height of the speed hump. **[1]**

(b) Show that $y = \frac{1}{5}(4x - 6x^2 + 4x^3 - x^4)$. **[3]**

(c) Find the area of the cross-section of the speed hump. **[7]**

8 **(i)** Show that $\int_0^2 (x^2 + 2x - 3)\,dx = \frac{2}{3}$. **[3]**

The diagram shows part of the curve $y = x^2 + 2x - 3$.



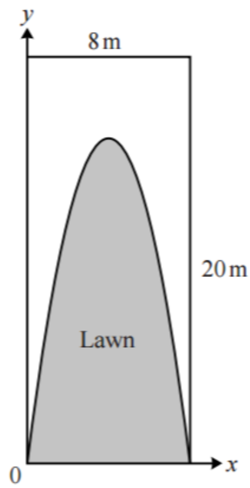
(ii) Marc claims that the total area between the curve, the x-axis and the lines $x = 0$ and $x = 2$ is $\frac{2}{3}$.

Explain why he is wrong. **[1]**

(iii) Calculate the total area between the curve, the x-axis and the lines $x = 0$ and $x = 2$. **[3]**

8 A mathematical gardener has a garden which is rectangular in shape measuring 20 metres by 8 metres. He wishes to arrange the garden so that approximately half of it is lawn and the rest flower bed.

He sets up a coordinate system as shown in the diagram below and maps out the graph of the curve $y = 8x - x^2$.



Show that the area of the lawn is approximately 53% of the total area. **[6]**

2 The gradient function of a curve that passes through the point (1, 2) is given by

$$\frac{dy}{dx} = 3x^2 - 4x + 7.$$

Find the equation of the curve. **[4]**

3 **(i)** Find the area enclosed between the curve $y = 8x^3$, the x-axis and the line $x = 2$. **[3]**

(ii) Hence, or otherwise, deduce the area between the x-axis, the y-axis, the line $x = 2$ and the curve $y = 8x^3 + 5$. **[1]**

4 **(i)** Find $\int_1^2 (x^2 + 2x + 3)\,dx$. **[4]**

(ii) Interpret your answer geometrically. **[1]**

11 Two curves, S_1 and S_2 have equations $y = x^2 - 4x + 7$ and $y = 6x - x^2 - 1$ respectively. The curves meet at A and at B.

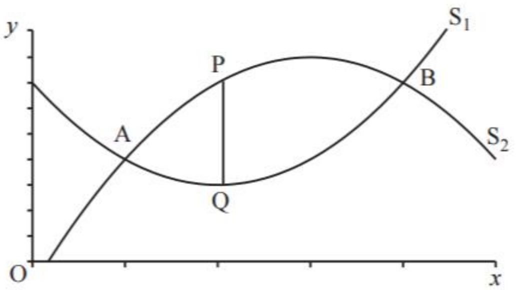


Fig. 11

(i) Show that the coordinates of A and B are (1, 4) and (4, 7) respectively. **[2]**

Points P and Q lie on S_2 and S_1 between A and B. P and Q have the same x coordinate so that PQ is parallel to the y-axis, as shown in Fig. 11.

(ii) Find an expression, in its simplest form, for the length PQ as a function of x. **[2]**

(iii) Use calculus to find the greatest length of PQ. **[4]**

(iv) Find the area between the two curves. **[4]**

7 The gradient function of a curve is given by $\frac{dy}{dx} = a + bx$. Find the values of a and b and the equation of the curve given that it passes through the points $(0, 2)$, $(1, 8)$ and $(-1, 2)$. [7]

8 A car moves in a straight line. Its velocity in metres per second, t seconds after passing a point A, is given by the equation

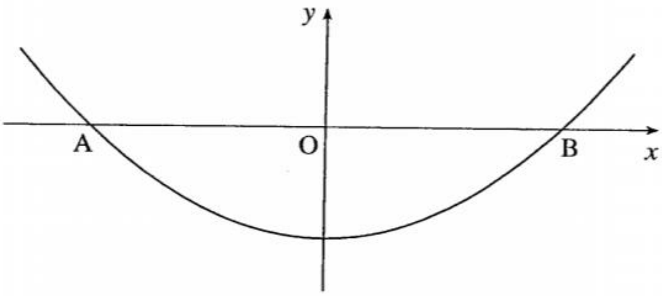
$$v = 27 - \tfrac{1}{8}t^3.$$

It comes to rest at a point B.

(i) Show that the car is at B when $t = 6$. [1]

(ii) Find the distance AB. [5]

13



The shape of the bed of a river is to be modelled mathematically. The diagram represents a cross-section of the river. A and B on the x-axis represent points on opposite banks of the river at water level. (Units are metres.)

The shape of the river bed between A and B is modelled by the equation

$$y = \tfrac{3}{16}(x^2 - 16).$$

(i) Find the coordinates of A and B and hence state the width of the river represented by the length AB. [2]

(ii) Find the depth of the river at its deepest point. [2]

(iii) Find the area of the cross-section of the river. [5]

(iv) The river flows at 20 metres a minute. You should assume that this rate applies to all points of this cross-section.

Find the volume of water that flows through this cross-section per minute. [1]

(v) Give two reasons why this model may not be a good model. [2]

1 Find $\int_1^3 (x^2 + 3)\,dx$. [4]

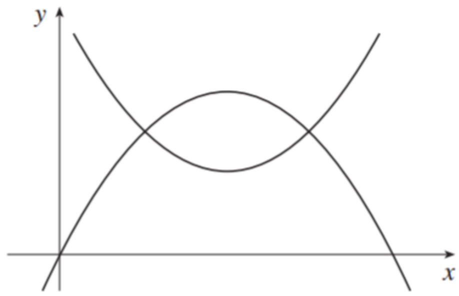
6 A curve has gradient given by $\frac{dy}{dx} = 2x + 2$. The curve passes through the point $(3, 0)$. Find the equation of the curve. [5]

2 A particle moves in a straight line. Its velocity, v m s^{-1}, t seconds after passing a point O is given by the equation

$$v = 6 + 3t^2.$$

Find the distance travelled between the times $t = 1$ and $t = 3$. [4]

8 The figure shows the graphs of $y = 4x - x^2$ and $y = x^2 - 4x + 6$.



(i) Use an algebraic method to find the x-coordinates of the points where the curves intersect. [3]

(ii) Calculate the area enclosed by the two curves. [4]

6 A speedboat accelerates from rest so that t seconds after starting its velocity, in $\mathrm{m\,s^{-1}}$, is given by the formula $v = 0.36t^2 - 0.024t^3$.

(i) Find the acceleration at time t. [3]

(ii) Find the distance travelled in the first 10 seconds. [4]

11 The side of a fairground slide is in the shaded shape as shown in Fig. 11. Units are metres.

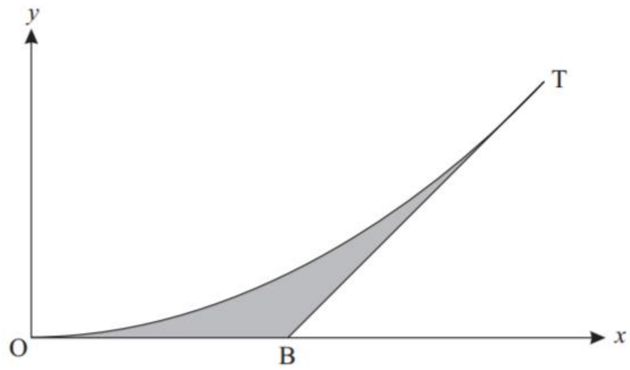


Fig. 11

The curve has equation $y = \lambda x^2$.

T has coordinates (4, 2). The line BT is a tangent to the curve at T. It meets the x-axis at the point B.

(i) Find the value of λ. [1]

(ii) Find the equation of the tangent BT and hence find the coordinates of the point B. [6]

(iii) Find the area of the shaded portion of the graph. [5]