

4. (a) Find the binomial expansion of

$$\sqrt[3]{(8 - 9x)}, \quad |x| < \frac{8}{9}$$

in ascending powers of x , up to and including the term in x^3 . Give each coefficient as a simplified fraction.

(6)

- (b) Use your expansion to estimate an approximate value for $\sqrt[3]{7100}$, giving your answer to 4 decimal places. State the value of x , which you use in your expansion, and show all your working.

(3)

$$\begin{aligned} & \sqrt[3]{8 - 9x} \\ &= 8^{\frac{1}{3}} \left(1 - \frac{9x}{8}\right)^{\frac{1}{3}} \\ &\approx 2 \left(1 + \frac{1}{3} \left(-\frac{9x}{8}\right) + \frac{\left(\frac{1}{3}\right) \left(-\frac{2}{3}\right) \left(-\frac{9x}{8}\right)^2}{2} + \frac{\left(\frac{1}{3}\right) \left(-\frac{2}{3}\right) \left(-\frac{5}{3}\right) \left(-\frac{9x}{8}\right)^3}{6}\right) \\ &= 2 - \frac{3x}{4} - \frac{9x^2}{32} - \frac{45x^3}{256} \end{aligned}$$

$$\begin{aligned} & \sqrt[3]{7100} \\ &= \sqrt[3]{1000 \times 7.1} \\ &= 10 \times \left(8 - 9 \times \frac{1}{10}\right)^{\frac{1}{3}} \\ &\approx 10 \left(2 - \frac{3}{4} \times \frac{1}{10} - \frac{9}{32} \times \left(\frac{1}{10}\right)^2 - \frac{45}{256} \times \left(\frac{1}{10}\right)^3\right) \\ &\approx 19.2201 \end{aligned}$$