OCR Additional Maths Exam Questions - Differentiation

2 Find the equation of the normal to the curve $y = x^3 + 5x - 7$ at the point $(1, -1)$. **[5]**

5 The curve $y = x^3 - 3x^2 - 9x + 7$ has two turning points, one of which is where $x = 3$.

(i) Find the coordinates of the other turning point and determine whether it is a maximum or minimum point. **[5]**

(ii) Sketch the curve. **[1]**

12 Fig. 12 shows the shape AOB that is to be made from card.

B is the point (5, 0) and OB is part of the curve with equation $y = 0.3x^2 - 1.5x$.

The line AB is the normal to the curve at B.

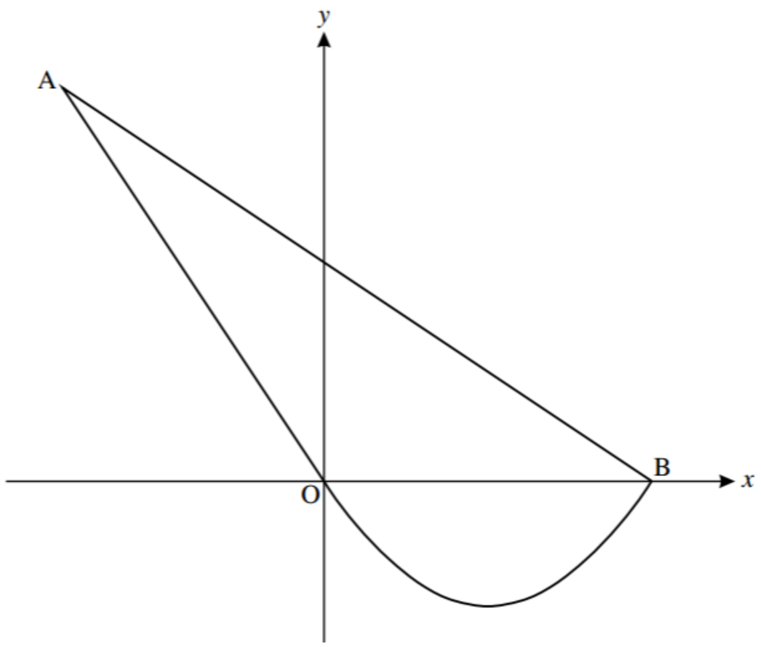


Fig. 12

(i) Find the equation of the line AB. **[4]**

The equation of the line AO is $2y + 3x = 0$.

(ii) Find the coordinates of the point A. **[3]**

(iii) Find the area of the shape AOB. **[5]**

2 The equation of a curve is $y = x^3 - x^2 - 2x - 3$.

Find the equation of the tangent to this curve at the point (3, 9). **[5]**

6 The equation of a curve is $y = 2x^3 - 9x^2 + 12x$.

(i) Show that the curve has a stationary point where $x = 2$. **[4]**

(ii) Determine whether the stationary value where $x = 2$ is a maximum or minimum. **[2]**

13 **(i)** Find the coefficients a, b and c in the expansion

$$(2 + h)^3 = 8 + ah + bh^2 + ch^3.$$ **[3]**

(ii) The graph of the equation $y = x^3$ passes through the points P and Q which have x-coordinates 2 and $2 + h$ respectively.

Show that the gradient of the chord PQ is $\frac{(2+h)^3 - 8}{h}$. **[3]**

(iii) Express $\frac{(2+h)^3 - 8}{h}$ as a quadratic function of h. **[2]**

(iv) As the value of h decreases, the point Q gets closer and closer to the point P on the curve. As h gets closer to 0 the chord PQ gets closer to being the tangent to the curve at P.

Deduce the value of the gradient of the tangent at P. **[1]**

(v) Kareen uses the same method to deduce the value of the gradient of the tangent at the point (2, 16) on the curve $y = x^4$.

The first three lines of her working are given below and in the answer booklet.

Take P to be the point (2, 16)

Take Q to be the point $(2 + h, (2 + h)^4)$

The gradient of the chord PQ is given by $\frac{(2+h)^4 - 16}{h} =$

Complete Kareen's working. **[3]**

5 **(i)** Use calculus to find the stationary points on the curve $y = x^3 - \frac{3}{2}x^2 - 6x + 3$. **[5]**

(ii) Sketch the curve on the axes provided showing the stationary points and the point where it cuts the y-axis. **[2]**

12 An object sinks through a thick liquid such that at time t seconds after being released on the surface the depth, s metres, is given by

$$s = 4t^2 - \frac{2t^3}{3} \qquad \text{for } 0 \leqslant t \leqslant 4.$$

(a) Find the formula for the velocity, v metres per second, t seconds after being released.
Hence show that the object stops sinking when $t = 4$. **[4]**

(b) Find

(i) the acceleration of the object when it is released on the surface of the liquid, **[4]**

(ii) the greatest depth of the object. **[2]**

(c) On the grids provided sketch the velocity-time and acceleration-time graphs. **[2]**

14 A curve has equation $y = 4x^3 - 5x^2 + 1$ and passes through the point A(1, 0).

(i) Find the equation of the normal to the curve at A. **[5]**

(ii) This normal also cuts the curve in two other points, B and C. Show that the x-coordinates of the three points where the normal cuts the curve are given by the equation $8x^3 - 10x^2 + x + 1 = 0$.
[2]

(iii) Show that the point B $\left(\frac{1}{2}, \frac{1}{4}\right)$ satisfies the normal and the curve. **[2]**

(iv) Find the coordinates of C. **[3]**

4 A train travels from station A to station B. It starts from rest at A and comes to rest again at B. The displacement of the train from A at time t seconds after starting from A is s metres where

$$s = 0.09t^2 - 0.0001t^3.$$

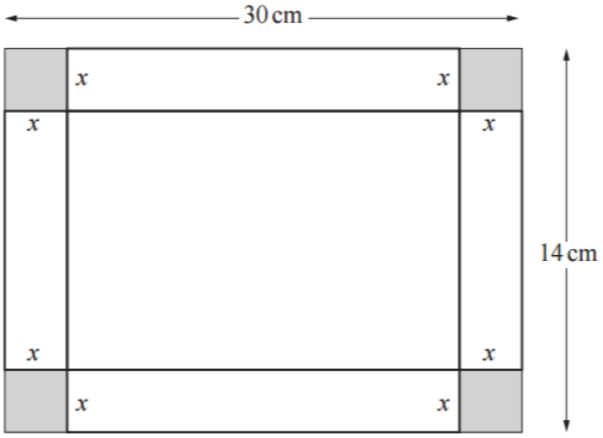
(i) Find the velocity at time t seconds after leaving A and hence find the time taken to reach B. Give the units of your answer. **[4]**

(ii) Find the distance between A and B. Give the units of your answer. **[2]**

10 **(i)** Find the coordinates of the point P on the curve $y = 2x^2 + x - 5$ where the gradient of the curve is 5.
[3]

(ii) Find the equation of the normal to the curve at the point P. **[3]**

11 Kala is making an open box out of a rectangular piece of card measuring 30 cm by 14 cm. She cuts squares of side x cm out of each corner and turns up the sides to form the box.



(i) Find an expression in terms of x for the volume, $V\text{cm}^3$, of the box and show that this reduces to $V = 4x^3 - 88x^2 + 420x$. **[4]**

(ii) Find the two values of x that give $\frac{dV}{dx} = 0$. **[5]**

(iii) Explain why one of these values should be rejected and find the maximum volume of the box using the other value. **[3]**

3 Find the equation of the tangent to the curve $y = x^3 + 3x - 5$ at the point (2, 9). **[5]**

5 A train accelerates from rest from a point O such that at t seconds the displacement, s metres from O, is given by the formula $s = \frac{3}{2}t^2 - 2t + 3$.

(i) Show by calculus that the acceleration is constant. **[3]**

(ii) Find the velocity after 5 seconds. **[2]**

11 Two curves, S_1 and S_2 have equations $y = x^2 - 4x + 7$ and $y = 6x - x^2 - 1$ respectively. The curves meet at A and at B.

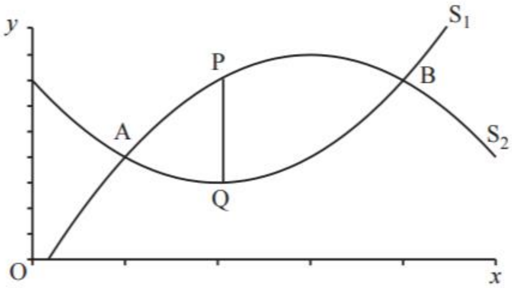


Fig. 11

(i) Show that the coordinates of A and B are (1, 4) and (4, 7) respectively. **[2]**

Points P and Q lie on S_2 and S_1 between A and B. P and Q have the same x coordinate so that PQ is parallel to the y-axis, as shown in Fig. 11.

(ii) Find an expression, in its simplest form, for the length PQ as a function of x. **[2]**

(iii) Use calculus to find the greatest length of PQ. **[4]**

(iv) Find the area between the two curves. **[4]**

1 Use calculus to show that there is a maximum point at $x = 3$ on the curve $y = 9x^2 - 2x^3$ and find the coordinates of this point. [5]

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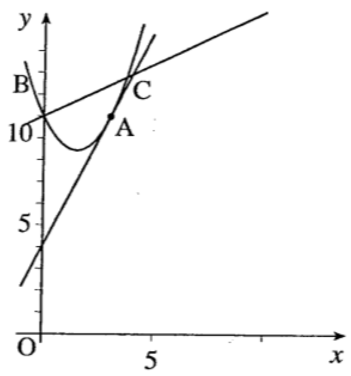

The curve shown has equation $y = \frac{2}{3}x^2 - 2x + 10$.

(i) Find the equation of the tangent to the curve at A (3, 10). [4]

(ii) Show that the equation of the normal to the curve at B (0, 10) is $2y - x = 20$. [3]

(iii) Find the coordinates of the point C where these two lines intersect. [3]

(iv) Calculate the length BC. [2]

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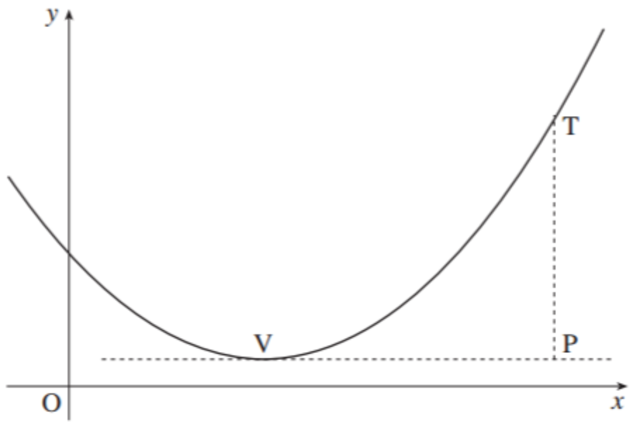

Fig. 14

Fig. 14 shows the quadratic curve $y = x^2 - 4x + 5$.

$\mathrm{V}(2, 1)$ is the minimum point of the curve.

$\mathrm{T}(5,10)$ is a point on the curve.

The line VP is the tangent to the curve at V and TP is perpendicular to this line.

(i) Write down the coordinates of P. [1]

(ii) Find the coordinates of M, the midpoint of VP. [2]

(iii) Find the equation of the tangent to the curve at T. [4]

(iv) Show that the tangent to the curve at T passes through the point M. [2]

(v) Use the result in part **(iv)** to suggest a way of drawing a tangent to a point on a quadratic curve without involving calculus. [3]

6 Find the equation of the tangent to the curve $y = x^3 - 3x + 4$ at the point $(2, 6)$. [4]

7 Use calculus to find the x-coordinate of the minimum point on the curve

$$y = x^3 - 2x^2 - 15x + 30.$$

Show your working clearly, giving the reasons for your answer. [7]

5 **(i)** Use calculus to find the stationary points on the curve $y = x^3 - 3x + 1$, identifying which is a maximum and which is a minimum. [6]

(ii) Sketch the curve. [1]

6 A speedboat accelerates from rest so that t seconds after starting its velocity, in m s^{-1}, is given by the formula $v = 0.36t^2 - 0.024t^3$.

(i) Find the acceleration at time t. [3]

(ii) Find the distance travelled in the first 10 seconds. [4]

11 The side of a fairground slide is in the shaded shape as shown in Fig. 11. Units are metres.

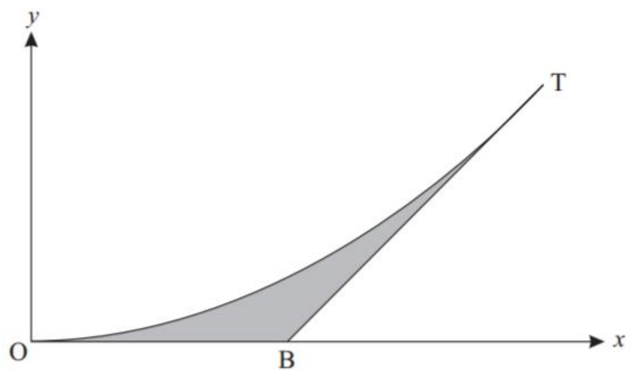

Fig. 11

The curve has equation $y = \lambda x^2$.

T has coordinates (4, 2). The line BT is a tangent to the curve at T. It meets the x-axis at the point B.

(i) Find the value of λ. [1]

(ii) Find the equation of the tangent BT and hence find the coordinates of the point B. [6]

(iii) Find the area of the shaded portion of the graph. [5]