

Core Pure 2 Series

Find the Maclaurin series for $\ln \frac{1+x}{1-x}$ and state its interval of validity.

By choosing a suitable value for x calculate the value of $\ln 5$, correct to 3 decimal places.

$$\ln \frac{1+x}{1-x} = \ln(1+x) - \ln(1-x)$$

$$= \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \right) - \left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots \right)$$

$$= 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + \dots + \frac{2}{2r+1}x^{2r+1}$$

$$-1 < x \leq 1$$

$$\frac{1+x}{1-x} = 5 \Rightarrow 1+x = 5-5x \Rightarrow x = \frac{2}{3}$$

$$\ln \frac{1+x}{1-x} \approx \frac{4}{3} + \left(\frac{2}{3}\right)^4 + \frac{2}{5}\left(\frac{2}{3}\right)^5 + \frac{2}{7}\left(\frac{2}{3}\right)^7 + \frac{2}{9}\left(\frac{2}{3}\right)^9 + \frac{2}{11}\left(\frac{2}{3}\right)^{11} + \frac{2}{13}\left(\frac{2}{3}\right)^{13} = 1.609 \text{ (3 d.p.)}$$