Core Pure 2 Methods in Differential Equations

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 169x^2 + 6$$

The auxiliary equation is $m^2 + 6m + 13 = 0$ the roots of which are $m = -3 \pm 2i$.

The solution of the homogeneous equation is $y = e^{-3x}(C\cos 2x + D\sin 2x)$

The complimentary function is of the form $y = Ax^2 + Bx + C$

$$y' = 2Ax + B$$

$$y'' = 2A$$

$$y'' + 6y' + 13y = 2A + 12Ax + 6B + 13Ax^2 + 13Bx + 13C$$

$$= 13Ax^2 + (12A + 13B)x + (2A + 6B + 13C) = 169x^2 + 6$$

$$A = 13 \qquad B = -12 \qquad C = \frac{6 - 26 + 72}{13} = 4$$

The general solution is $y = e^{-3}(C\cos 2x + D\sin 2x) + 13x^2 - 12x + 4$