

Core Pure 2 Differential Equations

$$\text{Given that } \frac{dx}{dt} = x - y, \quad \frac{dy}{dt} = 2x + 4y$$

and $x = -2$ and $y = 3$ when $t = 0$ express x and y in terms of t .

$$4y = \frac{dy}{dt} - 2x$$

$$4 \frac{dy}{dt} = \frac{d^2y}{dt^2} - 2 \frac{dx}{dt}$$

$$4 \frac{dy}{dt} = \frac{d^2y}{dt^2} - 2(x - y)$$

$$4 \frac{dy}{dt} = \frac{d^2y}{dt^2} - \left(\frac{dy}{dt} - 4y \right) + 2y$$

$$\frac{d^2y}{dt^2} - \frac{5dy}{dt} + 6y = 0$$

$$m^2 - 5m + 6 = 0$$

As a quick check on your working up to this point

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow m^2 - \text{Trace}(A)m + \text{Det}(A) = 0$$

$$m = 2 \text{ or } m = 3$$

$$y = Ae^{2t} + Be^{3t}$$

$$\frac{dy}{dt} = 2Ae^{2t} + 3Be^{3t} = 2x + 4(Ae^{2t} + Be^{3t})$$

$$x = -Ae^{2t} - \frac{B}{2}e^{3t}$$

When $t = 0$

$$x = -A - \frac{B}{2} = -2 \text{ and } y = A + B = 3$$

$$2A + B = 4$$

$$A = 1 \text{ and } B = 2$$

$$x = -e^{2t} - e^{3t}$$

$$y = e^{2t} + 2e^{3t}$$