

## Core Pure 2 Complex Numbers

Find the exact value of

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 \theta \, d\theta$$

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$$2 \cos \theta = e^{i\theta} + e^{-i\theta}$$

$$\begin{aligned} \cos^4 \theta &= \frac{1}{16} (e^{i\theta} + e^{-i\theta})^4 \\ &= \frac{1}{16} (e^{4i\theta} + 4e^{2i\theta} + 6 + 4e^{-2i\theta} + e^{-4i\theta}) \\ &= \frac{1}{16} (2 \cos 4\theta + 8 \cos 2\theta + 6) \end{aligned}$$

$$\begin{aligned} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 \theta \, d\theta &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{8} \cos 4\theta + \frac{1}{2} \cos 2\theta + \frac{3}{8} \, d\theta \\ &= \left[ \frac{1}{32} \sin 4\theta + \frac{1}{4} \sin 2\theta + \frac{3\theta}{8} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ &= \frac{3\pi}{8} \end{aligned}$$

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