Core Pure 2 Complex Numbers

Find the exact value of

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 \theta \, d\theta$$

$$2\cos\theta = e^{i\theta} + e^{-i\theta}$$

$$\cos^4\theta = \frac{1}{16}\left(e^{i\theta} + e^{-i\theta}\right)^4$$

$$= \frac{1}{16}\left(e^{4i\theta} + 4e^{2i\theta} + 6 + 4e^{-2i\theta} + e^{-4i\theta}\right)$$

$$\frac{1}{16}\left(2\cos 4\theta + 8\cos 2\theta + 6\right)$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 \theta \, d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8} \, d\theta$$

$$= \left[\frac{1}{32}\sin 4\theta + \frac{1}{4}\sin 2\theta + \frac{3\theta}{8}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{3\pi}{8}$$