Core Pure 1 Vectors

Find the shortest distance between the parallel lines with equations

$\boldsymbol{r} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k} + \lambda(-3\boldsymbol{i} - 4\boldsymbol{j} + 5\boldsymbol{k})$ and $\boldsymbol{r} = \boldsymbol{j} + \boldsymbol{k} + \mu(-3\boldsymbol{i} - 4\boldsymbol{j} + 5\boldsymbol{k})$

Consider the point $(2, -1, 1)$ on the first line and a general point $(-3\mu, 1 - 4\mu, 1 + 5\mu)$ on the second line. The square of the distance between the points is

$$(2 + 3\mu)^2 + (-2 + 4\mu)^2 + (5\mu)^2 = 50\mu^2 - 4\mu + 8$$

$$\frac{d}{d\mu}(50\mu^2 - 4\mu + 8) = 100\mu - 4 \Rightarrow$$

$\mu = \frac{1}{25}$ when the distance is a minimum.

The minimum distance = $\sqrt{50\left(\frac{1}{25}\right)^2 - \frac{4}{25} + 8} = 2.81$ (3 s.f.)