Core Pure 1 Series

Show that $\displaystyle\sum_{i=1}^{n}\sum_{r=1}^{i}(r^2+r)=\frac{1}{12}n(n+1)(n+2)(n+3)$

Hence show that $\displaystyle\sum_{j=1}^{n}\sum_{i=1}^{j}\sum_{r=1}^{i}r=\frac{1}{24}n(n+1)(n+2)(n+3)$

$$\sum_{r=1}^{i}(r^2+r)=\frac{1}{6}i(i+1)(2i+1)+\frac{1}{2}i(i+1)$$

$$=\frac{1}{6}i(i+1)(2i+4)$$

$$=\frac{1}{3}i(i+1)(i+2)$$

$$\frac{1}{3}\sum_{i=1}^{n}(i^3+3i^2+2i)$$

$$=\frac{1}{3}\sum_{i=1}^{n}i^3+\sum_{i=1}^{n}i^2+\frac{2}{3}\sum_{i=1}^{n}i$$

$$=\frac{1}{12}n^2(n+1)^2+\frac{2}{12}n(n+1)(2n+1)+\frac{4}{12}n(n+1)$$

$$=\frac{1}{12}n(n+1)(n(n+1)+2(2n+1)+4)$$

$$=\frac{1}{12}n(n+1)(n^2+5n+6)$$

$$=\frac{1}{12}n(n+1)(n+2)(n+3)$$

$$\sum_{r=1}^{i}r=\frac{i}{2}(i+1)\Rightarrow$$

$$\sum_{j=1}^{n}\sum_{i=1}^{j}\sum_{r=1}^{i}r=\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{j}(i^2+i)=\frac{1}{24}n(n+1)(n+2)(n+3)$$

Bury Maths Tutor