

Core Pure 1 Linear Transformations

$$M = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$

The point  $(a, b, c)$  maps to the point  $(3, 2, 1)$  under  $M$ .

Find  $M^{-1}$  and hence find the values of  $a, b$  and  $c$ .

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$$|M| = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 2 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = -1 - 1 + 6 = 4$$

$$C = \begin{pmatrix} \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} -1 & -1 & 3 \\ 3 & -1 & -1 \\ -1 & 3 & -1 \end{pmatrix}$$

$$C^T = \begin{pmatrix} -1 & 3 & -1 \\ -1 & -1 & 3 \\ 3 & -1 & -1 \end{pmatrix}$$

$$M^{-1} = \frac{1}{4} \begin{pmatrix} -1 & 3 & -1 \\ -1 & -1 & 3 \\ 3 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{1}{4} \begin{pmatrix} -1 & 3 & -1 \\ -1 & -1 & 3 \\ 3 & -1 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{3}{2} \end{pmatrix}$$

$$a = \frac{1}{2} \quad b = -\frac{1}{2} \quad c = \frac{3}{2}$$