Find a vector perpendicular to both $\begin{pmatrix} 1\\3\\7 \end{pmatrix}$ and $\begin{pmatrix} 2\\1\\5 \end{pmatrix}$ without using the vector product.

Let the vector be
$$\begin{pmatrix} 1\\ a\\ b \end{pmatrix}$$

 $\begin{pmatrix} 1\\ 3\\ 7 \end{pmatrix} \cdot \begin{pmatrix} 1\\ a\\ b \end{pmatrix} = 1 + 3a + 7b = 0$
 $\begin{pmatrix} 2\\ 1\\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1\\ a\\ b \end{pmatrix} = 2 + a + 5b = 0$
 $3a + 7b = -1$
 $a + 5b = -2$
 $a = \frac{9}{8}$ $b = -\frac{5}{8}$
 $\begin{pmatrix} 1\\ 9\\ \end{pmatrix}$

A vector perpendicular to both given vectors is $\begin{pmatrix} \frac{1}{9} \\ \frac{9}{8} \\ -\frac{5}{8} \end{pmatrix}$ or $\begin{pmatrix} 8 \\ 9 \\ -5 \end{pmatrix}$

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