C4 Parametric Equations

The curve C has parametric equations

$x = 2\cos t\,,\;\; y = \sqrt{3}\cos 2t, \qquad 0 \le t \le \pi$

where t is a parameter.

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

The point P lies on C where $t = \frac{2\pi}{3}$
The line l is a normal to C at P.

(b) Show that an equation for l is

$2x - 2\sqrt{3}y - 1 = 0$

(c) The line l intersects the curve C again at the point Q.

Find the exact coordinates of Q.

You must show clearly how you obtained your answers.

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$\frac{dy}{dt} = -2\sqrt{3}\sin 2t, \qquad \frac{dx}{dt} = -2\sin t$ and $\frac{dy}{dx} = \frac{2\sqrt{3}\cos t \sin t}{\sin t} = 2\sqrt{3}\cos t.$

At P $\frac{dy}{dx} = 2\sqrt{3}\cos\frac{2\pi}{3} = -\sqrt{3}$, the gradient of the normal is $\frac{1}{\sqrt{3}}$,

$x = 2\cos\frac{2\pi}{3} = -1$ and $y = \sqrt{3}\cos\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

The equation of the normal is $y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1)$

Multiplying by $2\sqrt{3}$ gives $2\sqrt{3}y + 3 = 2x + 2$

$2x - 2\sqrt{3}y - 1 = 0$

Substituting $x = 2\cos t$ and $y = \sqrt{3}(2\cos^2 t - 1)$ into the equation of the normal gives

$4\cos t - 6(2\cos^2 t - 1) - 1 = 0$

$12\cos^2 t - 4\cos t - 5 = 0$

$(6\cos t - 5)(2\cos t - 1) = 0$

$\cos t \ge 0$ when $0 \le t \le \pi$ therefore $\cos t = \frac{5}{6}$, $x = \frac{5}{3}$ and $y = \sqrt{3}\left(2 \times \left(\frac{5}{6}\right)^2 - 1\right) = \frac{7\sqrt{3}}{18}$

Q is the point $\left(\frac{5}{3}, \frac{7\sqrt{3}}{18}\right)$.

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