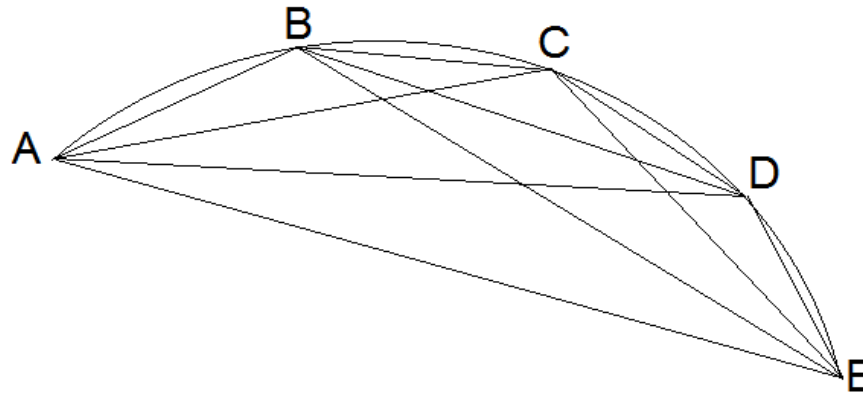


BMO 1966 question 4

The points A, B, C and D are four consecutive vertices of a regular polygon.

If $\frac{1}{AB} = \frac{1}{AC} + \frac{1}{AD}$ how many sides must the polygon have?



Let $AB=1$, $AC=\alpha$, $AD=\beta$ and $AE=\gamma$.

A line from a vertex to the next vertex but one has length α , a line from a vertex to the next vertex but two has length β and so on.

If $\frac{1}{AB} = \frac{1}{AC} + \frac{1}{AD}$ then $1 = \frac{1}{\alpha} + \frac{1}{\beta}$ and so $\alpha\beta = \alpha + \beta$.

By Ptolemy's theorem, applied to quadrilateral ABCE, $\alpha\beta = \alpha + \gamma$ and so $\beta = \gamma$.

A line from a vertex to the next vertex but two is equal in length to a line from a vertex to the next vertex but three. In the diagram, $BE=AE$ and so the polygon has two further vertices.

The polygon has seven sides.