An A Level Maths Question on Logs

Show that $\log_a b = \dfrac{\log_c b}{\log_c a}$

$$\log_a b = k \Rightarrow a^k = b \Rightarrow k \log_c a = \log_c b \Rightarrow k = \frac{\log_c b}{\log_c a} \Rightarrow \log_a b = \frac{\log_c b}{\log_c a}$$

Hence show that $\log_a b = \dfrac{1}{\log_b a}$

$$\frac{1}{\log_b a} = \frac{\log_b b}{\log_b a} = \log_a b$$

Solve $\log_3 x = \log_x 7$

$$\log_3 x = \log_x 7 \Rightarrow \log_x 3 = \frac{\log_3 7}{\log_3 x} \Rightarrow (\log_3 x)^2 = \log_3 7 \Rightarrow \log_3 x = \pm\sqrt{\log_3 7} \Rightarrow x = 3^{\pm\sqrt{\log_3 7}}$$

Bury Maths Tutor