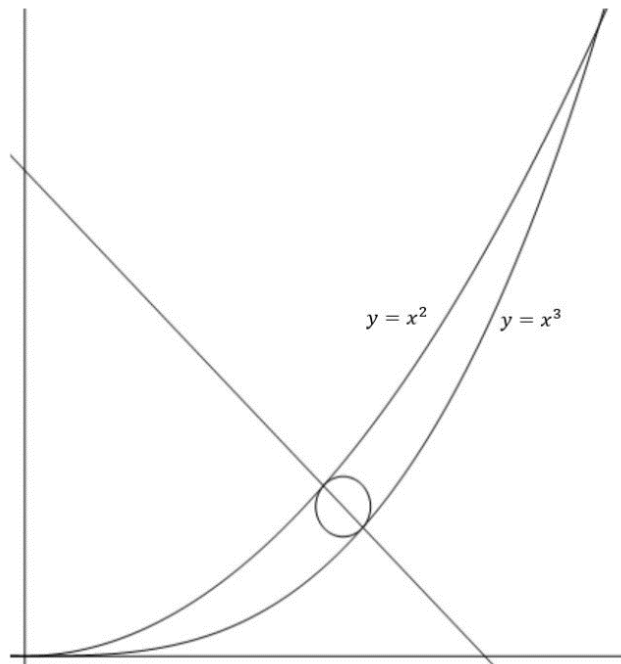


### An A level maths question



The straight line is a diameter of the circle and is normal to both curves. Find the area of the circle.

---

#### Solution

Let the points of interest be  $(a, a^2)$  and  $(b, b^3)$ .

The gradients of the curves at these points are equal and the line is perpendicular to the curves at these points so  $2a = 3b^2 = \frac{b-a}{a^2-b^3}$ .

$$3b^2 \left( \left( \frac{3b^2}{2} \right)^2 - b^3 \right) = b - \frac{3b^2}{2}$$

$$27b^6 - 12b^5 = 4b - 6b^2$$

$$b \neq 0$$

$$27b^5 - 12b^4 + 6b - 4 = 0$$

Using the Newton-Raphson method:

$$b_{n+1} = b_n - \frac{27b_n^5 - 12b_n^4 + 6b_n - 4}{135b_n^4 - 48b_n^3 + 6}$$

Starting with  $b_0 = 0.5$  gives  $b_1 = 0.607407\dots$ ,  $b_2 = 0.589537\dots$ ,  $b_3 = 0.588700\dots$ ,  $b_4 = 0.588698\dots$ .

$$b = 0.5886989388, \quad a = \frac{3b^2}{2} = 0.5198496608.$$

The area of the circle is  $\pi \frac{(b-a)^2 + (b^3 - a^2)^2}{4} \approx 0.007167$  units<sup>2</sup>.