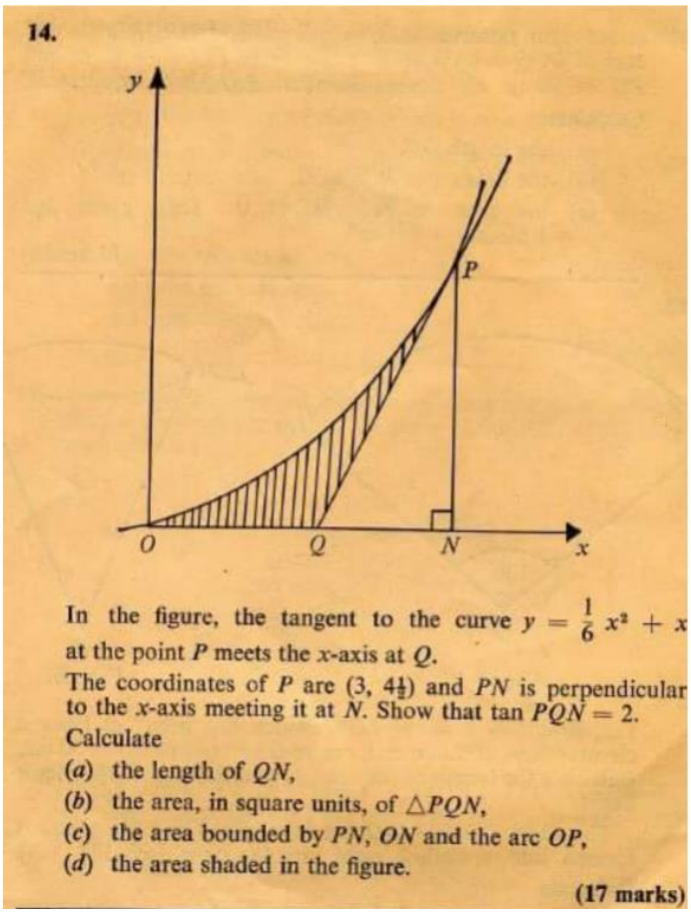
A 1976 O level mathematics question



$\frac{dy}{dx} = \frac{1}{3}x + 1.$

At P, $\frac{dy}{dx} = \frac{1}{3} \times 3 + 1 = 2.$

QN is therefore $\frac{1}{2} \times \frac{9}{2} = \mathbf{\frac{9}{4}}.$

The area of triangle PQN is $\frac{1}{2} \times \frac{9}{4} \times \frac{9}{2} = \mathbf{\frac{81}{16}}.$

The required area is $\int_0^3 \left(\frac{x^2}{6} + x\right) dx = \frac{3^3}{18} + \frac{3^2}{2} = \frac{27}{18} + \frac{9}{2} = \frac{3}{2} + \frac{9}{2} = \mathbf{6}.$

The area shaded is $6 - \frac{81}{16} = \frac{96-81}{16} = \mathbf{\frac{15}{16}}.$

Bury Maths Tutor