<u>AQA S2 Continuous Random Variables</u>

7 Engineering work on the railway network causes an increase in the journey time of commuters travelling into work each morning.

The increase in journey time, T hours, is modelled by a continuous random variable with probability density function

$$f(t) = \begin{cases} 4t(1-t^2) & 0 \leqslant t \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $E(T) = \frac{8}{15}$. *(3 marks)*

(b) (i) Find the cumulative distribution function, $F(t)$, for $0 \leqslant t \leqslant 1$. *(2 marks)*

(ii) Hence, or otherwise, for a commuter selected at random, find

$$P(\text{mean} < T < \text{median})$$ *(5 marks)*

(a)

$$E(T) = \int_{-\infty}^{\infty} t\,f(t)dt = \int_0^1 t \times 4t(1-t^2)dt$$

$$= \int_0^1 (4t^2 - 4t^4)\,dt = \left[\frac{4t^3}{3} - \frac{4t^5}{5}\right]_0^1$$

$$= \frac{4}{3} - \frac{4}{5} = \frac{8}{15}$$

(b) (i)

$$F(\tau) = \int_{-\infty}^{\tau} f(t)dt = \int_0^{\tau}(4t - 4t^3)dt = 2\tau^2 - \tau^4$$

$$F(t) = 2t^2 - t^4 \qquad 0 \le t \le 1$$

$$F(t) = P(T \le t)$$

(ii)

$P(\text{mean}<\text{T}<\text{median}) = \text{P(T}<\text{median)-P(T}<\text{mean)}$

$= F(\text{median})\text{-}F(\text{mean})$

$$= 0.5 - F\left(\frac{8}{15}\right) = 0.5 - \left(2\left(\frac{8}{15}\right)^2 - \left(\frac{8}{15}\right)^4\right) = 0.012$$