

AQA FP2 2017

1 (a) Express $\frac{r+1}{(2r+1)(2r+3)}$ in partial fractions.

[2 marks]

(b) Use the method of differences to find $\sum_{r=1}^n \frac{(-1)^{r+1}(r+1)}{(2r+1)(2r+3)}$.

[3 marks]

$$\frac{r+1}{(2r+1)(2r+3)} \equiv \frac{A}{2r+1} + \frac{B}{2r+3} \Rightarrow r+1 \equiv A(2r+3) + B(2r+1) \Rightarrow$$

$$\text{Putting } r = -\frac{1}{2} \text{ leads to } \frac{1}{2} \equiv 2A \Rightarrow A = \frac{1}{4}$$

$$\text{and putting } r = -\frac{3}{2} \text{ leads to } -\frac{1}{2} \equiv -2B \Rightarrow B = \frac{1}{4}$$

$$\frac{r+1}{(2r+1)(2r+3)} \equiv \frac{1}{4(2r+1)} + \frac{1}{4(2r+3)}$$

$$\sum_{r=1}^n \frac{(-1)^{r+1}(r+1)}{(2r+1)(2r+3)} \equiv \sum_{r=1}^n \frac{(-1)^{r+1}}{4(2r+1)} + \frac{(-1)^{r+1}}{4(2r+3)}$$

$$\equiv \frac{1}{12} + \frac{1}{20} - \frac{1}{20} - \frac{1}{28} \dots \frac{(-1)^n}{4(2n-1)} + \frac{(-1)^n}{4(2n+1)} + \frac{(-1)^{n+1}}{4(2n+1)} + \frac{(-1)^{n+1}}{4(2n+3)} \equiv \frac{1}{12} + \frac{(-1)^{n+1}}{4(2n+3)}$$