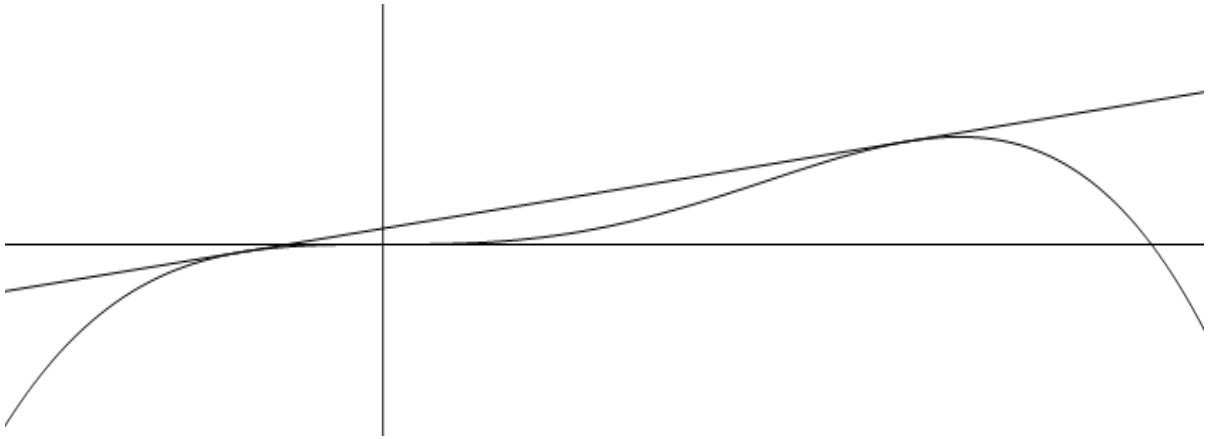
A question for A level mathematics students



The curve shown above has equation $y = x^3 - x^4$. The straight line is a tangent to the curve at two points. Find the equation of the line.

Solution

If the equation of the line is $y = ax + b$ then $ax + b - x^3 + x^4 \equiv (x - c)^2(x - d)^2$. Comparing coefficients:

$$2c + 2d - 1 = 0 \Longrightarrow c + d = \frac{1}{2}$$

$$c^2 + 4cd + d^2 = 0 \Longrightarrow (c + d)^2 + 2cd = 0 \Longrightarrow cd = -\frac{1}{8}$$

$$2cd^2 + 2c^2d + a = 0 \Longrightarrow a = -2cd(c + d) = \frac{1}{8}$$

$$b - c^2d^2 = 0 \Longrightarrow b = \frac{1}{64}$$

The equation of the line is $y = \frac{x}{8} + \frac{1}{64}$.

Maths notes and resources